Chapter 2

The mathematical model of bipolar choice and its testing

A choice between two alternatives, one of which is linked to a positive pole and the other to a negative pole, is called *bipolar choice*. For example, a choice between truth and lie, or between the adjectives ‘beautiful’ and ‘ugly’ in characterizing an object is called *bipolar*. A bipolar choice may be complete, such as when an alternative is chosen, or incomplete, such as when a score is marked on a scale (for example, a given object is evaluated as 0.7 good and 0.3 bad). Experiments show that the mean frequency of complete choices of poles is approximately equal to the mean score on the interval [0,1]. So, we will consider a score on a scale as the probability that the subject is ready to choose the positive pole.

2.1. The mathematical model

Let us construct a mathematical model of bipolar choice (Lefebvre, 2004, 2006b). We will assume that a living organism can be represented as

$$B = \psi(z,S), \quad (2.1.1)$$

where B corresponds to the organism’s behavior, z - to the influence of the environment, and S is an internal variable representing the organism’s mental experience.

If we consider S as a basic variable and z as a parameter, then (2.1.1) can be written as

$$B = \Phi_z(S). \tag{2.1.2}$$

For bipolar choice we must connect B with the probability of choosing one of the poles, for example, the positive pole, and z with probability of the environment's pressure toward the positive pole at the moment of making choice, x_1. Then we can write (2.1.2) as

$$X(S) = F_{x_1}(S)\,. \tag{2.1.3}$$

We assume that $X(S)$ is a differentiable function, where $0<X(S)<1$, $0< x_1 <1$, $S \geq 0$ and

$$X(0) = x_1. \tag{2.1.4}$$

Condition (2.1.4) means that at S=0 the probability of choosing the positive pole is equal to the environment's pressure toward the positive pole at the moment of making the choice.

Suppose that it takes a certain time to make the choice. If the value of S is constant, the probability of choosing the positive pole is X_S. However, if S increases by some small value ΔS during this interval, then the choice procedure becomes different: we call this the *axiom of repeated choice*.

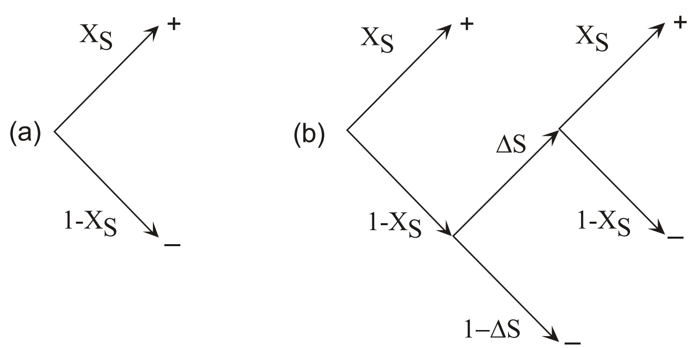

Fig. 2.1.1. Choice trees: (*a*) for the value of the internal variable equal to S; (*b*) for the value of the internal variable equal to $S+\Delta S$

First, the subject makes a choice with probability X_S of choosing the positive pole. If the positive pole is chosen, the subject executes his choice. If the negative pole is chosen, the subject cancels his choice with the small probability ΔS and repeats the procedure of making choice. The result of the repeated choice is then executed. The two trees in Fig.2.1.1 depict the axiom of repeated choice for S and $S+\Delta S$.

In accordance with tree (*b*),

$$X_{S+\Delta S} = X_S + (1 - X_S)\Delta S X_S \quad . \tag{2.1.5}$$

Now we will search for a differentiable function $X(S)$ represented as

$$X(S+\Delta S) = X(S) + (1 - X(S))\Delta S X(S) + o(\Delta S) \quad .$$

After transformations and passage to the limit at $\Delta S \to 0$ we obtain the differential equation

$$\frac{dX(S)}{dS} = (1 - X(S))X(S) \quad . \tag{2.1.6}$$

Solving it using (2.1.4) as initial condition, we obtain the logistic function

$$X(S) = \frac{x_1}{x_1 + (1 - x_1)e^{-S}} \, . \tag{2.1.7}$$

Henceforth we will write this expression as

$$X_1 = \frac{x_1}{x_1 + x_2 - x_1 x_2} \, , \tag{2.1.8}$$

where

$$x_2 = e^{-S} \, . \tag{2.1.9}$$

The value of the internal variable S is interpreted as the level of importance, for the subject, of choosing the positive pole. With constant x_1, the greater S, the greater X_1.

Let us examine (2.1.9). We interpret the value of x_2 as the subject's estimation of the probability with which the world inclines him toward choice of the positive pole. It is important to emphasize the difference between x_1 and x_2: x_1 is the world's objective pressure to choose the positive pole, and x_2 is a subjective evaluation of that pressure. Also, x_1 is a *local* characteristic of the environment at the *given moment*, and x_2 is a *global* characteristic based on the subject's experience in the presence and past. It is a measure of the world's positivity for the subject.

We assume also that the subject has an image of the self, X_2, which is the subject's estimation of the probability of his choosing the positive pole. While X_1 is an objective characteristic that belongs to the *real* world, X_2 is a subjective characteristic belonging to the mental world.

The subject with image of the self can be represented as a box with input x_1 and output X_1, into which another box has been inserted with input x_2 and output X_2 (Fig. 2.1.2).

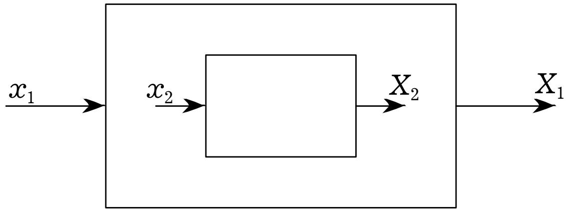

Fig. 2.1.2. Subject with image of the self

We assume that

$$\frac{X_1}{x_1} = \frac{X_2}{x_2}. \tag{2.1.10}$$

In other words, the subject's image of the self is similar to the subject, in the sense that they equally amplify an input signal. It follows from (2.1.8) and (2.1.10) that

$$X_2 = \frac{x_2}{x_1 + x_2 - x_1 x_2}. \tag{2.1.11}$$

Expressions (2.1.8) and (2.1.11) play an important role in our further considerations.

The concept of 'image' can be generalized by assuming that the image of the self may have an image of the self and that the latter also has an image of the self etc. We assume that expression (2.1.10) holds for each image and its image of the self. Fig. 2.1.3 depicts the subject with a chain of images:

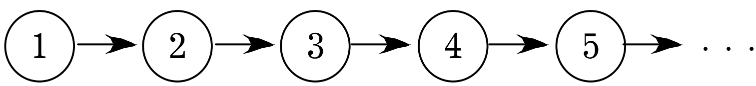

Fig. 2.1.3. Subject with chain of images: 1- subject, 2 - subject's image of the self, 3 - image of the self of the image of the self, etc.

The arrows depict the relation "knows"; the statement "1 knows 2,who knows 3,... who knows *n*" corresponds to the entire chain.

Let us introduce the relation "is aware of," equivalent to double relation "knows": if *A* "knows" *B*, and *B* "knows" *C*, then *A* "is aware of" *C*. We depict this relation with a curved arrow.

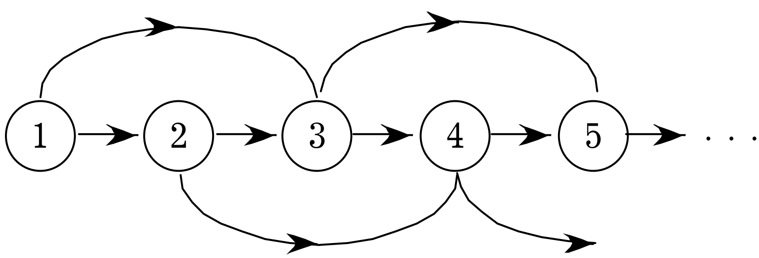

Fig. 2.1.4. Subject with chain of images and relations: "knows" and "is aware of"

Subject 1 is aware of image 3, image 2 is aware of image 4, image 3 is aware of image 5, etc. We call a subject or image from which an arrow originates an *original*, and one to which an arrow points a *copy*. We say that a copy is equivalent to the original if they are characterized by the same pair x, X, where x is the probability of pressure toward the positive pole, and X is the probability of choosing the positive pole. Let us introduce a postulate of equivalency:

In awareness, the original and the copy are equivalent.

It follows from this postulate that elements with odd numbers are characterized by a pair x_1, X_1, and those with even numbers by a pair x_2, X_2, given by the expressions (2.1.8) and (2.1.11). This correspondence is depicted in Fig. 2.1.5.

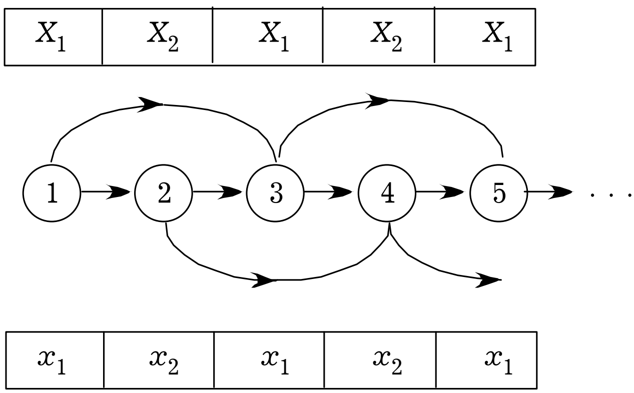

Fig. 2.1.5. Values of pairs corresponding to the subject and his images of the self

The scheme in Fig. 2.1.5 corresponds to the mathematical model of bipolar choice; it generates two pairs of distributions:

$$(X_1, 1-X_1), (x_1, 1-x_1) \quad \text{and} \quad (X_2, 1-X_2), (x_2, 1-x_2) .$$

After conducting the necessary empirical examination, we will demonstrate that these distributions can be formally represented as the product of work by a chain of heat engines which can perform functions different from those in "real life," namely, they can generate probability distributions. Therefore, unlike quantum mechanics, where the probability distribution is found with a Fourier series, in our model the probability distribution is found through representing the subject's mental domain as a chain of heat engines (Chapter 3).

2.2. Categorizing stimuli

Categorization is evaluation of a stimulus over a set of intensity levels. For example:

very weak *weak* *moderate* *strong* *very strong*

Usually, numbers are used instead of verbal evaluations:

1 2 3 4 5

The procedure is as follows. First, the subject is presented with the weakest and strongest stimuli to be used in a particular experiment, for example, two bars of 5 cm and 105 cm; after that, all other stimuli are presented one by one. The subject's task is to assign each stimulus to a certain category. The data obtained allow construction of a graph linking the physical intensity of the stimuli with their categorical estimation. Researchers were surprised by the initial results. They expected to find a linear relation between physical measures and categorical estimations of such qualities as length, area, and duration. Instead, they found curved graphs.

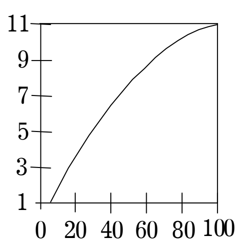

Fig. 2.2.1. Estimation of the length of metal bars.
Length in centimeters is mapped onto the X-axis, and categorical estimation onto the Y-axis (data by Stevens & Galanter, 1957)

For example, the graph in Fig. 2.2.1 shows the results of an experiment on length estimation.

The experiments that followed demonstrated that the graph's curvature depends on the distribution of the intensity of stimuli. If, in a given experiment, there are more weak stimuli, the curvature increases; if there are more strong stimuli, the curvature decreases (Fig. 2.2.2).

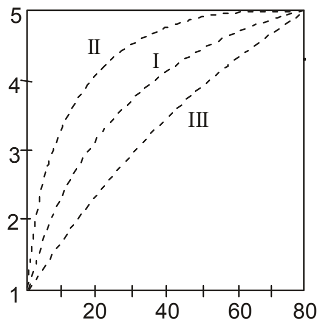

Fig. 2.2.2. Distribution of stimuli: I - weak and strong stimuli presented equally; II - weak stimuli presented more often; III - strong stimuli presented more often (Stivens & Galanter, 1957)

A problem of categorical graph curvature has puzzled researchers for more than sixty years, but no truly convincing explanation has been found. We will reexamine this problem from the point of view of the formal model of bipolar choice (Fig. 2.1.2). Let us introduce the following parallels:

x_1 is the normalized physical intensity of a presented stimulus,

x_2 is the mean normalized intensity of all stimuli presented to the subject,

X_1 is the normalized categorical estimation of a given stimulus, in the following called simply "categorical estimation."

In the mathematical scheme of bipolar choice, the process of categorization is given by the function

$$X_1 = \frac{x_1}{x_1 + x_2 - x_1 x_2} \quad . \tag{2.2.1}$$

The choice is not complete. Variable x_1 represents perception, x_2 represents memory, and X_1 is the outcome of the stimulus estimation.

Consider the experiment with categorization of bar length. First, two bars are presented: the shortest (5 cm) and the longest (105 cm). The former acquires the role of a negative pole, and the latter that of a positive pole. The stimulus intensity is the length of the bar. From the semantic point of view, the bar's length equates to its endowment with positive value, because in the construct *long-short*, the adjective *long* plays the role of the positive pole, and *short* that of the negative pole.

At the level of perception, the subject's cognitive system determines the value

$$x_1 = \frac{\psi - \psi_{\min}}{\psi_{\max} - \psi_{\min}} , \qquad (2.2.2)$$

where ψ is the physical length of the bar currently presented, $\psi_{\max}$ that of the longest bar, and $\psi_{\min}$ that of the shortest bar in the series. The value of x_1 is the pressure toward the positive pole at the level of perception. Then, the subject's cognitive system finds the mean value of the intensity of the stimuli previously presented:

$$x_2 = \frac{x_1^{(1)} + x_1^{(2)} + \ldots + x_1^{(n)}}{n} . \qquad (2.2.3)$$

The value of x_2 represents the class of stimuli shown to the subject. It is the level of the world's positivity in the context of a given series of estimations. Obtaining the values for x_1 and x_2 from (2.2.2) and (2.2.3) and substituting them into (2.2.1), we find the categorical estimation for a given bar.

Let us consider a set of experiments with different values of x_2. For each x_2, there is a curve corresponding to function X_1. The smaller the value of x_2, the more convex the curve (see Fig. 2.2.3).

When the intensities of stimuli are shifted toward the weakest stimulus, the mean intensity decreases, and *for that reason* the curve representing the data is more convex. When the intensities of stimuli are shifted toward the strongest one, the mean intensity increases, and *for that reason* the curve representing those data is less convex.

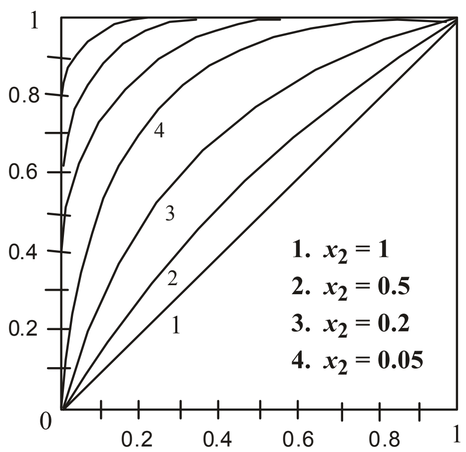

Fig. 2.2.3. Family of curves corresponding to function X_1 for different values of x_2

Therefore, our model explains the phenomena appearing in experiments with categorization.

2.3. The Golden section phenomenon in choice and categorization

In the mid 1950s, George Kelly (1955) proposed the hypothesis that, in each person's cognitive sphere, there exists a unique system of bipolar category constructs such as *active-passive, sharp-blunt, strong-weak,* etc. This system allows an individual to have a multivariate view of other people's qualities and personalities. According to Kelly, the constructs inherent to an individual are bipolar, with positive and negative poles that are used with equal frequency if the person evaluates a large number of other people. Kelly's disciples, however, Adams-Webber and Benjafield, found (1973), that equal estimations are very rare and that the frequency of choosing the positive pole is 0.62. They hypothesized that the theoretical value of the frequency of using the positive pole in evaluating others is the famous golden section $(\sqrt{5}-1)/2 = 0.618...$, a ratio associated with beauty and attractiveness from antiquity to the present (Benjafield, & Adams-Webber, 1976).

Having analyzed numerous experiments with bipolar estimations, we found that the golden section ratio is indeed present both in evaluating persons and in evaluating inanimate objects (Lefebvre, 1985, 1987, 1992). It appears precisely in those cases when the subjects do not have operational means for isolating the quality they must evaluate.

For example, let a subject is given a picture of a flower and a scale for evaluation of the flower's beauty (Fig. 2.3.1).

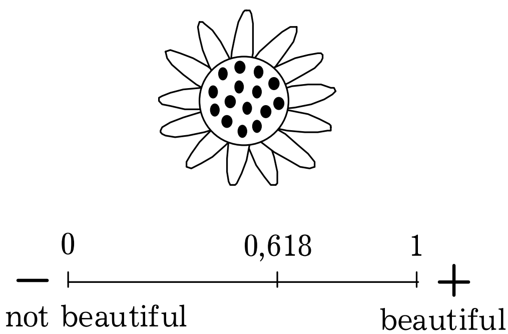

Fig. 2.3.1. Stimuli categorization

We will try to understand the structure of the computation performed by the subject's cognitive system in such cases. First, it polarizes the scale: "beautiful" becomes the positive pole, and "not beautiful" becomes the negative pole. Second, the subject's cognitive system provides values for the variables x_1 and x_2. Since beauty does not have direct physical intensity, we may suppose that x_1 takes on a neutral value

$$x_1 = \frac{1}{2}\,. \tag{2.3.1}$$

For the same reason, we assume that the value of x_2 is not a physical measure. It is equal to the score of the flower's beauty generated by the subject:

$$x_2 = X_1\,. \tag{2.3.2}$$

The flower, for the subject, is both the world and, at the same time the object of evaluation. By substituting the values of (2.3.1) and (2.3.2) into the expression

$$X_1 = \frac{x_1}{x_1 + (1 - x_1)x_2}\,, \tag{2.3.3}$$

we obtain the following equation

$$X_1^2 + X_1 - 1 = 0\,. \tag{2.3.4}$$

Its positive root is the golden section:

$$X_1 = \frac{\sqrt{5} - 1}{2} = 0.618\ldots\,. \tag{2.3.5}$$

This is the probability with which the subject is ready to choose the positive pole in evaluating the flower.

We decided to test this prediction experimentally and empirically.

Experiment with pinto beans

The experimental material consisted of fifty small transparent envelopes, each containing two pinto beans. The beans were chosen to have as few differences as possible. The subject's task was to pick up the envelopes one by one, evaluate a pair as good or bad, and drop the envelope into one of two boxes marked "+" and "-". The ratio of positive estimations was 0.611 (V.D. Lefebvre, 1990). This experiment supports the hypothesis that the theoretical value of positive evaluation in the absence of an operational criterion is equal to the golden section, i.e., to the predicted value. When the experiment with pinto beans was replicated, the ratio of positive estimation was equal to 0.64, insignificantly different from the golden section value (Anderson and Grice, 2009).

Moral judgment

Independently of our research, McGraw (1985) conducted the following experiment. Each subject was given a scenario describing either a good action or a bad action. For example, someone found a wallet and returned it to its owner; a blind girl dropped money and someone helped her to collect it: these are good actions. Or, the wallet was not returned to the owner; the blind girl was not helped to collect her money: these are bad actions. The task was to predict the percentage of students who would act in the way described by a given scenario. The mean estimation by those who received good scenarios was 62%, as opposed to 39% by those who received bad ones.

We can explain these results as follows. For all subjects the positive pole is "100% good students" and the negative pole is "0% good students". Thus, for all subjects the mean score is close to the golden section value, 62% and 61%. The subjects who received the bad scenarios marked the percentage of bad students, 39%, as the instructions required. In this experiment, $x_1 = \frac{1}{2}$, because pressures toward both poles were the same, and $x_2 = X_1$, i.e., the subjects did not have previous experience of similar experiments. Under these conditions, equation (2.3.3) turns into (2.3.4), whose positive root is the golden section value.

Mere exposure

The phenomenon of the golden section manifests itself in the choice between two objects that are barely distinguishable, provided that one of them is associated with a positive pole and the other with a negative pole. To polarize the objects, one of them is shown beforehand, making it more likely to become the positive pole. To avoid determining in advance the value of x_2 (which is related to previous conscious experience), the exposure must be subliminal.

Let us look at one such experiment conducted without any relationship to our model (Kunst-Wilson & Zajonc, 1980). Twenty improper octagons were used as the experimental material. For each subject, the twenty octagons were divided into two groups of ten. In the first part of the experiment, the subject was shown ten octagons from one of the groups five times each with an exposure time of one millisecond. This short exposure time does not allow the subject to perceive the object consciously. In the second part of the experiment, the subject was presented with a series of pairs of octagons, each consisting of one octagon previously shown and one new. The experimenter did not inform the subjects that the previously shown figures were among the octagons presented in the second part of the experiment. The task was to choose, in each pair, the octagon that the subject "likes more." The subjects chose the one presented in the first part of the experiment with a frequency of 0.60.

Below are frequencies of choosing the alternatives presented in advance, obtained from other similar experiments:

Seamon et al. (1983)	0.61
Mandler et al. (1987)	0.62
Bonano et al. (1986)	0.66; 0.63; 0.62; 0.61; 0.63; 0.62

We see that the frequencies are grouped around 0.62 (a fact that the experimenters apparently did not notice). The preliminary exposure of one of the alternatives polarized the pairs in which it would subsequently appear; the one that had been shown previously became the positive pole, and the other, the new one, the negative pole. Attractiveness is not here a measurable quality, so that $x_1 = \frac{1}{2}$. The stimuli used in the experiment had not previously been evaluated by the subjects, so that the subjects had no history of evaluating them. This explains the appearance of the golden section ratio.

Medians in referenda

The mathematical model of bipolar choice predicts a hitherto unremarked phenomenon involving the results of referenda. For example, the following draft bill is offered to voters:

Proposed, to reduce the number of optional subjects in public schools, in order to better teach required subjects, on the one hand, and to facilitate balancing the state budget, on the other.

YES __________ NO __________

Fig. 2.3.2. The draft bill

Let us assume that the formulation of this question in a given social context polarizes the alternatives “yes” and “no” in such way that for the majority of voters one of them becomes the positive pole and the other the negative pole. For example, let’s say that shortly before the referendum there was an active campaign in favor of broad education in public schools. Thus, the role of positive pole is played by the alternative “no”.

At the moment of casting a ballot, the words “yes” and “no” are equal; thus, $x_1 = \frac{1}{2}$. An ordinary voter has no in-depth experience in educational policy; thus $x_2 = X_1$, and the phenomenon of the golden section should appear. Therefore, under ideal conditions, people should choose the positive pole with a frequency of 0.62. In reality, however, there are voters who think about the issue and make their decision in advance, not at the moment of casting their ballot. We may think there are not too many of them, but there are enough to decrease or increase the frequency of the positive choice relative to 0.62. Further, we presume that:

- decreases or increases in the frequency of choosing the positive pole in relation to 0.62 happen equally often on the set of all referenda and
- in each referendum, the frequency of choosing the positive pole does not drop below 0.5 .

Under these assumptions, *the median distribution* of winning poles must be equal to the golden section value, i.e., the number of polls winning with lower than 62% votes must be equal to the number of polls winning with higher than 62% votes.

We have analyzed all the referenda in California from 1884 to 1990 (Eu, 1983a,b; 1985a,b; 1987; 1989a,b,c).

Figure 2.3.3 (next page) represents the distribution for referenda analyzed. Its median is equal to 62%.

A similar analysis was conducted for referenda in Switzerland 1886-1978 (Butler & Ranney, 1978). The median found was 63%. We also analyzed Oregon referenda 1904-1914 (Barnett, 1915) and found the median equal to 62%. Finally, we used the results of US referenda for 1977-1988 (Ranney, 1978; 1981; 1983; 1985; 1987; 1989). The median again was equal to 62%.

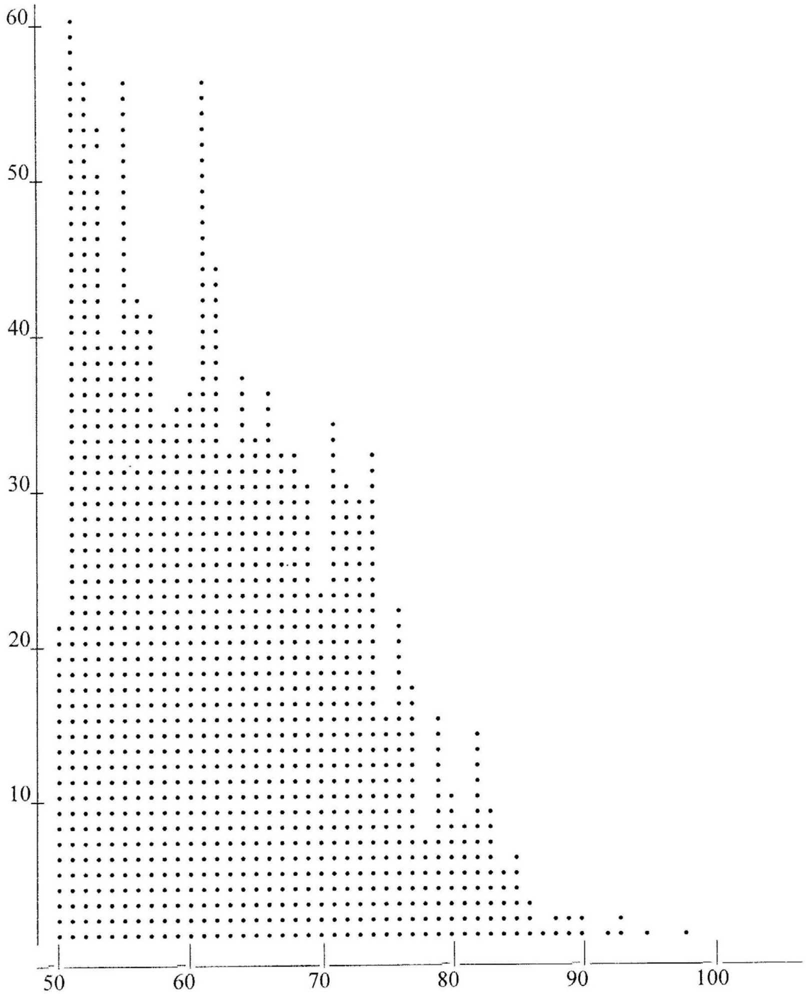

Fig. 2.3.3. Distribution of winning polls in California referenda 1884-1990. Each dot represents one referendum. The horizontal axis is the percentage scale 50-100%. Each column of dots shows the number of polls won with the given percentage of votes.

The two-humped graph

Let us consider an experiment by Poulton and Simmonds (1985). The subjects' task was to evaluate the degree of lightness of a sample of gray paper situated between black and white papers. The tone of the gray sample was picked in such a way that, on a psychological scale, it was exactly in the middle between the tones of the black and white samples. Each subject had to mark his evaluation of the gray sample's lightness on a hundred-millimeter scale, one end of which corresponded to black and the other to white. Only the very first touch of the subject's pencil was taken into account. The result of the experiment is given in Fig. 2.3.4; it is a two-humped graph with a dip in the middle.

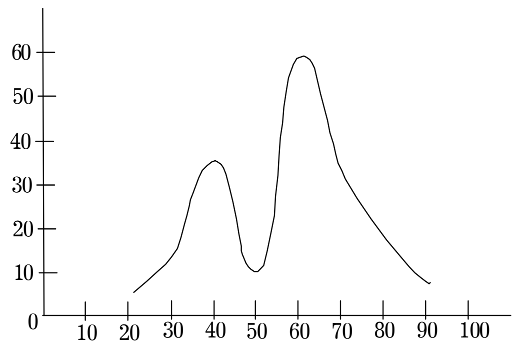

Fig. 2.3.4. A two-humped graph obtained in the experiment with lightness estimation

Psychophysics cannot explain this phenomenon, but we can explain it using the mathematical model of bipolar choice. Suppose that one portion of the subjects took the white sample as the positive pole, and the other part took the black sample as positive. Thus, the first group was evaluating the degree of whiteness of the gray sample, and the second group was evaluating the degree of its blackness. Since the pressures toward the two poles were equal,

$x_1 = \frac{1}{2}$. The subjects did not have experience in similar estimations (only the first touch was counted), so $x_2 = X_1$. Therefore, in both groups the estimation was equal to the golden section value. The peaks of the humps are located at 62 mm from the white end and 62 mm from the black end of the scale.

2.4. Altruism

It has been known for centuries that people who have committed some act they repent of are more inclined toward altruism. In the twentieth century, this opinion has been supported experimentally. Moreover, a related phenomenon was discovered. It turned out that people become more inclined to altruism not only when they feel guilty of something, but also as a result of observing the world's unfairness.

Here is one of the experiments in question (Regan, 1971). The student-subjects were asked to give a slight electric shock to a rat. Unnoticeably for the subject, the experimenter increased the amperage so that the rat jumped. The subjects were divided into three groups. In the first group, the amperage was not changed. In the second group, the amperage was increased but the subjects were told that they were not at fault, it was a short circuit. In the third group, after the rat jumped, the subjects were told that they had made a mistake, and that the experiment would have to be stopped.

Subsequently, each subject was involved in a situation in which he had the opportunity to donate small amount of money to a summer student project. The subjects from the second and third groups donated money significantly more often than those from the first group.

Let us consider this experiment from the point of view of the mathematical model of bipolar choice.

1. There are two alternatives: to donate money or not to donate money.

2. Variable x_1 is the pressure on the subject at the moment of the request for a donation; variable x_2 is the level of the world's positivity at that moment; X_1 is the probability with which the subject will choose the positive pole; X_2 is the subject's estimation of his readiness to choose the positive pole.
3. These variables are connected with the following expressions:

$$X_1 = \frac{x_1}{x_1 + x_2 - x_1 x_2}, \qquad (2.4.1)$$

$$X_2 = \frac{x_2}{x_1 + x_2 - x_1 x_2}. \qquad (2.4.2)$$

In the first group, the values of x_1 and x_2 are not changed in the course of the experiment. In the second group, the value of x_2 changes after the subject sees the rat suffering; it decreases, because the world has become less positive. It follows from (2.4.1) that, with the value of x_1 remaining constant and x_2 decreasing, the value of X_1 increases monotonically. Thus, the subjects should become more altruistic, as was found in the experiment.

In the third group, the experimenter influenced the subject's image of the self. It follows from (2.4.1) and (2.4.2) that

$$X_1 = 1 - (1 - x_1) X_2. \qquad (2.4.3)$$

With a constant value of x_1 we can consider X_2 an independent variable (such that variable x_2 becomes dependent). The feeling of guilt means decreasing the degree of positivity of one's image of the self, i.e., decreasing the value of X_2. It follows from (2.4.3) that with decreasing X_2, X_1 increases, i.e., the subject demonstrates greater altruism.

We see that the mathematical model of bipolar choice explains the increase of altruism resulting both from observing the world's unfairness and from feeling one's own guilt.

2.5. Bipolar choice in birds and animals

Almost a half-century ago, Richard Herrnstein published the results of his experiments with birds (1970). A pigeon was placed in a Skinner chamber with two feeders; by pecking the feeder, the bird received a small ration of grain. The feeders were controlled by independent reinforcement programs that the experimenter could vary. Under the influence of these programs, one might suppose that the pigeon's behavior could be shaped in different ways, but that was not the outcome. The pigeons used one particular strategy: their frequency of responses was approximately proportional to the frequency of reinforcement. This finding was called the Matching Law. Subsequent experiments showed systematic deviations from proportionality: birds were pecking one of the feeders more often than the Matching Law predicted. Moreover, it was found in many experiments that pigeons and rats turn to the "poorer" feeder more often than proportionality would require. Various other correlations were suggested, and, finally, William Baum (Baum et al., 1999) offered a new formula producing good predictions:

$$\frac{B_2}{B_1} = b\frac{r_2}{r_1}, \qquad (2.5.1)$$

where B_1 is the frequency of going to one feeder; B_2 is the frequency of going to the other feeder; r_1 is the frequency of reinforcements given in the first feeder, and r_2 the frequency given in the second. We can always choose the feeders' numeration such that $b \leq 1$ holds.

It should be noted that only recently has the model of bipolar choice begun to be used in research on animal choice (Lefebvre & Sanabria, 2008). Let us write (2.5.1) as

$$\frac{1 - B_1}{B_1} = b\frac{1 - r_1}{r_1} \qquad (2.5.2)$$

and then rewrite it as

$$B_1 = \frac{r_1}{r_1 + b - r_1 b}\,. \tag{2.5.3}$$

We see that (2.5.3) coincides with the expression

$$X_1 = \frac{x_1}{x_1 + x_2 - x_1 x_2}\,, \tag{2.5.4}$$

which shows the probability of choosing the positive pole in the mathematical model of bipolar choice. This fact suggests that animals, as well as humans, use binary valuations ‘*good-bad*’.

This problem is described in more detail in Appendix III.