Chapter 3.
The physical model and its testing

In the previous chapter we presented a number of empirical and experimental arguments supporting the contention that the mathematical model of bipolar choice, as set by the correlations

$$X_1 = \frac{x_1}{x_1 + x_2 - x_1 x_2}, \tag{3.1}$$

$$X_2 = \frac{x_2}{x_1 + x_2 - x_1 x_2}, \tag{3.2}$$

manifests itself in human and animal choice. We hypothesized that consciousness is an ideal physical process. In our further considerations, we will assume that expressions (3.1) and (3.2) are related not only to the behavior of living creatures but to their mental experience as well. In this chapter, we will search for ideal physical processes that are described by those correlations.

3.1. Heat engine

Consider an abstract heat engine (Fig. 3.1.1).

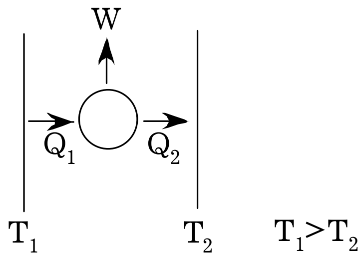

Fig. 3.1.1. Abstract heat engine

The engine takes heat Q_1 from a hot reservoir with temperature T_1, produces work W, and gives heat Q_2 to a cold reservoir with temperature T_2. The functioning of the engine obeys the first and second laws of thermodynamics:

I. The law of conservation of energy:

$$Q_1 = Q_2 + W \; ; \tag{3.1.1}$$

II. The law of undiminished entropy:

$$H_2 \geq H_1 \, , \tag{3.1.2}$$

where

$$H_1 = \frac{Q_1}{T_1} \tag{3.1.3}$$

is the decrease of entropy resulting from taking heat Q_1 from the hot reservoir, and

$$H_2 = \frac{Q_2}{T_2} \tag{3.1.4}$$

is the increase of entropy resulting from returning heat Q_2 to the cool reservoir.

The following values will be used in our discussion.

Entropy change:

$$\Delta H = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} \; . \tag{3.1.5}$$

Efficiency:

$$\rho_1 = \frac{Q_1 - Q_2}{Q_1} \; . \tag{3.1.6}$$

Maximal efficiency:

$$\rho_0 = \frac{T_1 - T_2}{T_1} \; . \tag{3.1.7}$$

Relative efficiency:

$$\omega_1 = \frac{\rho_1}{\rho_0} . \tag{3.1.8}$$

Lost available work:

$$\Delta W_1 = T_2\left(\frac{Q_2}{T_2} - \frac{Q_1}{T_1}\right) . \tag{3.1.9}$$

The main concepts of thermodynamics are described in Appendix I.

Compensation process

For reversible heat engines, the change of entropy is equal to zero, and the lost available work is also equal to zero. For non-reversible heat engines, the lost available work is greater than zero.

Let us include one more engine between the hot reservoirs 2 and 3 with temperatures T_2 and T_3, where

$$\frac{T_2}{T_3} = \frac{T_1}{T_2} . \tag{3.1.10}$$

Engine 2 takes from reservoir 2 heat equal to that given to it by engine 1 and produces work equal to the lost available work of engine 1. In this way, engine 2 compensates for the "imperfection" of engine 1 (Fig. 3.1.2).

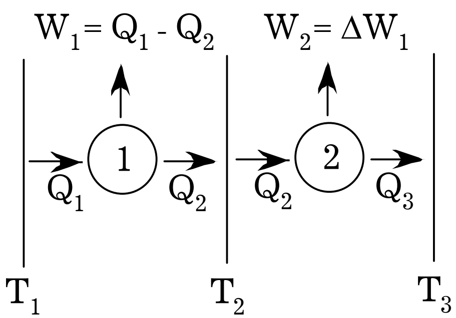

Fig. 3.1.2. Engine 2 compensates for the imperfection of engine 1

Engine 2's efficiency is

$$\rho_2 = \frac{\Delta W_1}{Q_2} = \frac{T_2\left(\frac{Q_2}{T_2} - \frac{Q_1}{T_1}\right)}{Q_2} \ , \tag{3.1.11}$$

and its relative efficiency is

$$\omega_2 = \frac{\rho_2}{\rho_0} . \tag{3.1.12}$$

It follows from (3.1.6) and (3.1.11) that

$$\frac{T_1 - T_2}{T_1} = \rho_1 + \rho_2 - \rho_1\rho_2 \tag{3.1.13}$$

and

$$\omega_1 = \frac{\rho_1}{\rho_1 + \rho_2 - \rho_1\rho_2} , \tag{3.1.14}$$

$$\omega_2 = \frac{\rho_2}{\rho_1 + \rho_2 - \rho_1\rho_2} . \tag{3.1.15}$$

Expressions (3.1.14) and (3.1.15) coincide with (3.1) and (3.2), which correspond to the mathematical model of bipolar choice if we assume that

$$\begin{aligned} \rho_1 &= x_1, \\ \rho_2 &= x_2, \\ \omega_1 &= X_1, \\ \omega_2 &= X_2. \end{aligned} \tag{3.1.16}$$

We can now write:

$$\frac{T_1 - T_2}{T_1} = x_1 + x_2 - x_1 x_2 \ , \tag{3.1.17}$$

$$\omega_1 = \frac{x_1}{x_1 + x_2 - x_1 x_2}, \qquad (3.1.18)$$

$$\omega_2 = \frac{x_2}{x_1 + x_2 - x_1 x_2}. \qquad (3.1.19)$$

The first engine stands for the subject, and the second for the subject's image of the self. Thus, we have found an ideal physical process related to the mental process of bipolar choice (Lefebvre, 1997, 2006a).

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The mathematical model of bipolar choice contains the following function:

$$x_2 = e^{-S}, \qquad (i)$$

where x_2 is the value of the world's positivity in the subject's mental world and S is the internal variable representing the degree of importance, for the subject, of choosing the positive pole.

Let us find an analogue for variable S in the heat engine model. Since $x_2 = \rho_2$, we can use (3.1.11) and write:

$$x_2 = \frac{\frac{Q_2}{T_2} - \frac{Q_1}{T_1}}{\frac{Q_2}{T_2}}. \qquad (ii)$$

The right side of this expression is the normalized value of the first engine's change of entropy; we designate it E. The x_2 can be written as

$$x_2 = e^{-\ln\frac{1}{E}}. \qquad (iii)$$

By comparing (iii) and (i) we see that

$$S = \ln \frac{1}{E} . \qquad \text{(iv)}$$

Thus, in the heat model, the internal variable S is the logarithm of the inverse normalized value of entropy change. The lesser E, the greater S.

Heat unit

Generally speaking, the second engine in Fig. 3.1.2 is also non-reversible. It also loses available work, and we can put a third engine after it, and then a fourth one, and so forth.

Consider a sequence of heat reservoirs whose temperatures decrease in geometrical progression. Between every two reservoirs we place an engine, so that every subsequent engine takes from its hot reservoir the heat returned to it by the preceding engine and produces the work equal to the available work lost by the preceding engine. Such a sequence is shown in Fig. 3.1.3.

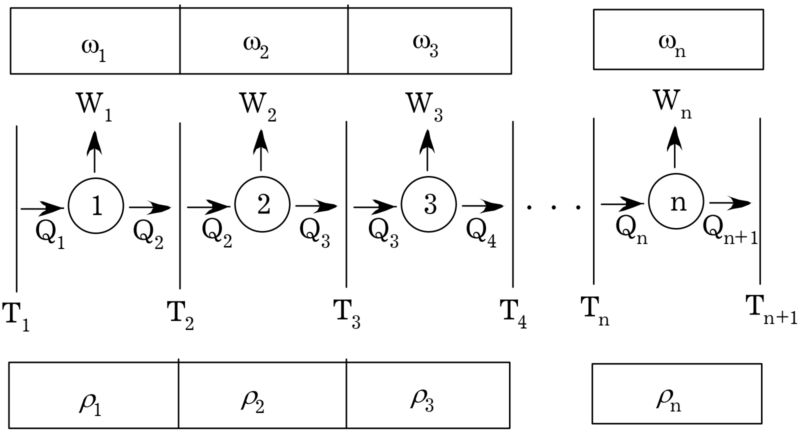

Fig. 3.1.3. Heat unit with header and footer

In addition, this unit is supplied with a header and footer; for each engine there is one corresponding cell on both header and footer. On the lower cells, the efficiencies are printed, and on the upper cells the relative efficiencies.

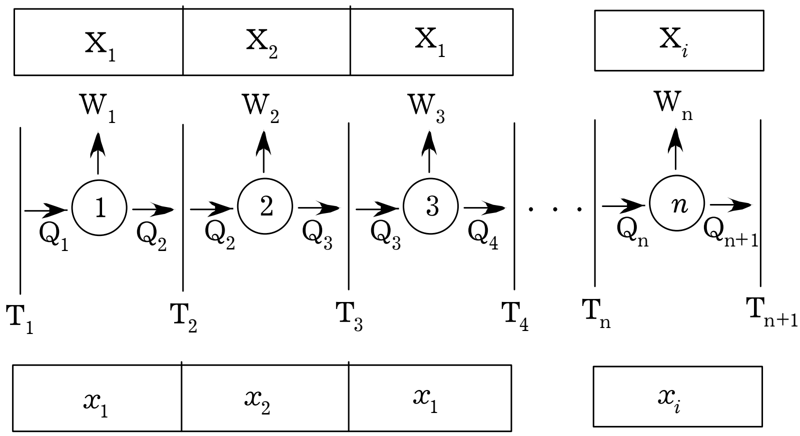

Fig. 3.1.4. Unit generating the mathematical model of bipolar choice (for odd n, $i = 1$; for even n, $i = 2$)

The following statement holds:

Sequences ρ_n and ω_n are periodic: for engines with odd numbers, $\rho_n=\rho_1$ and $\omega_n=\omega_1$, and for engines with even numbers, $\rho_n=\rho_2$ and $\omega_n=\omega_2$ (Lefebvre, 1997).

Using (3.1.16), we can depict the sequence of engines as shown in Fig. 3.1.4, where X_1 and X_2 are linked with x_1 and x_2 by expressions (3.1) and (3.2).

We see that the unit generates the scheme given in Fig. 2.1.5, which corresponds to the mathematical model of bipolar choice, and that the alternation of variables on the headers and footers happens automatically without recourse to the postulate of equivalency (see Chapter 2).

Therefore, the unit in Fig. 3.1.4 has a psychological interpretation. It depicts the subject who has an image of the self, which, in turn, has an image of the self, which also has an image of the self, and so forth. The analogy between the heat unit and the hierarchy of images may be established by two ways. In the first way, which we call *descending reflexion*, the first, leftmost engine represents the subject; the second engine is the subject's image of the self; the third engine is the image of the self's image of the self, etc.

In the second way, called *ascending reflexion*, the engine with number n, the rightmost engine, represents the subject; the engine with number n-1 is the subject's image of the self; the engine n-2 is the image of the self's image of the self, etc. Under descending reflexion, the images go in the order of decreasing temperature; under ascending reflexion, the images go in the order of increasing temperature[1].

The first step has been taken. We have found a formal connection between the mathematical model of bipolar choice and an ideal physical process which obeys two fundamental laws: the first and second laws of thermodynamics. This process cannot be situated in the physical brain of human beings or animals. It is possible that we may shed light on the work of the *eidos*-navigator (see Conclusion). Now, we have a hypothesis to test.

3.2. Two laws of psychophysics

Our task is to find out if there are psychological phenomena which cannot be explained by the mathematical model of bipolar choice but can be explained by the physical model constructed in the previous section. In this section, we will demonstrate that the main laws of classical psychophysics - the Weber-Fechner law (Fechner, 1860) and the Stevens law (1975) - can be deduced from our physical model. In addition, with the help of this model we will deduce the set of harmonical intervals in music.

The research into mathematical correlations between the physical characteristics of stimuli and their subjective perception began in the nineteenth century. Gustav Fechner, relying on Weber's experiments and ideas, demonstrated that the ratio of intensities of psychological perception is proportional to the ratio of logarithms of physical intensities. For example, if the physical weights of three

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[1] This definition of descending and ascending reflexion seems more natural than the one given in my book *Cosmic Subject*.

pieces are related as 2:8:16, the intensities of their psychological feelings are related as 1:3:4, that is, the ratio between physical stimuli diminishes in perception: the third weight is eight times heavier than the first one, but a person feels that it is only four times heavier. The Weber-Fechner law can be written as follows:

$$\psi_F = c \log \frac{\varphi}{\varphi_0}, \qquad (3.2.1)$$

where φ is the physical intensity of a stimulus, φ_0 the threshold value at which the stimulus begins to be perceived, ψ_F the psychological evaluation of the stimulus, and c the coefficient of proportionality.

Fechner's contemporaries were greatly impressed by his discovery of the logarithmic law. It seemed that a law linking the physical world with the human mental domain had finally been found. Some people believed that this law was as fundamental as Newton's law of universal gravitation. Fechner was sure that his law would never be disproven, if only, as he said, because scientists would never agree on how to refute it. He was right only in part.

In 1961, Stanley Stevens published a paper entitled "To Honour Fechner and Repeal His Law." This paper was the result of Stevens' long-term research. He conducted his experiments differently than Fechner: Fechner's subjects had to distinguish very close stimuli, while Stevens' subjects evaluated the intensities of very different stimuli. Moreover, Fechner's subjects were not trained in advance; they evaluated stimuli by intuition, whereas Stevens' subjects were carefully taught how to evaluate stimuli correctly. Stevens' subjects' evaluations followed the power law:

$$\psi_S = cS^{\beta}, \qquad (3.2.2)$$

where S is the intensity of the physical stimulus, ψ_S is the subject's evaluation of the stimulus physical intensity, and β is a parameter depending on the type of stimulus. For example, for loudness, $\beta = 0.67$, for weight $\beta = 1.45$; c is the coefficient of proportionality.

We will demonstrate that both laws are embedded in the physical model, and that it clarifies the conditions under which these laws reveal themselves. The Weber-Fechner law appears in experiments where the subjects evaluate the intensities of their feelings in relation to stimuli, while the Stevens law holds where the subjects directly evaluate the physical intensity of a stimulus.

Let us begin our deduction of the laws of psychophysics. First of all, we will introduce theoretical analogues to the perceptive inputs to the physical model. These provide a connecting link between the physical world and the human mental domain. We will then show that the role of perceptive inputs can be played by the cylinders of heat engines. To deduce the laws of psychophysics, we will need three one-cylinder engines containing an ideal gas (Fig. 3.2.1).

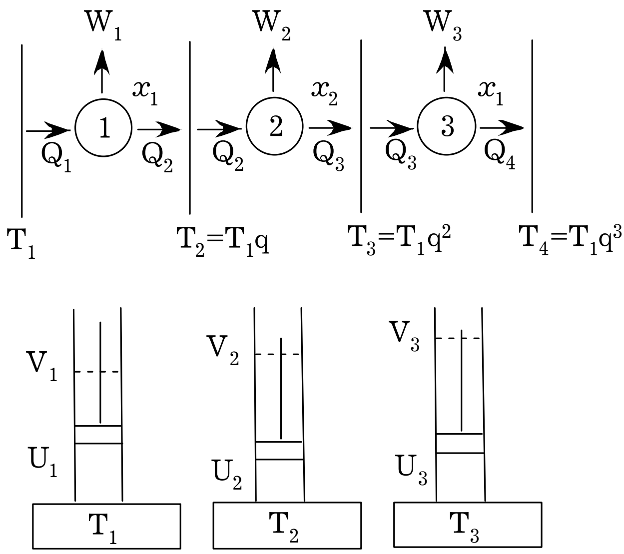

Fig. 3.2.1. Unit corresponding to the subject making psychological evaluations (q is a positive number less than 1)

We will consider two subjects with descending and ascending reflexion, respectively. In descending reflexion, the first engine corresponds to the subject, the second engine to the subject's image of the self, and the third engine to the image of the self's image. The latter is the subject's *cognizant* image of the self: the subject knows that this is not he himself "in reality," but his image of the self. In ascending reflexion, the third engine corresponds to the subject, the second to the subject's image of the self, and the first to the subject's cognizant image. Work W_1, W_2, and W_3 performed by the engines are interpreted as the subjects' inner feelings.

For the subject with descending reflexion, W_1 is his immediate feelings, W_2 his feeling of his feelings, and W_3 his cognizant feeling. For the subject with ascending reflexion, the immediate feeling is W_3; W_2 is his feeling of that feeling, and W_1 is the subject's cognizant feeling.

We assume that the engines work cyclically. At the beginning of a cycle, the gas has the temperature of the heat source. The gas takes some heat from this source and expands isothermally. The details of how the engine returns the heat to the cold reservoir are not essential. The state of the ideal gas is described by the following equation:

$$PV = RT \ , \tag{3.2.3}$$

where P is pressure, V volume, T temperature, and R the gas constant. Assume that $R = 1$. While expanding isothermally from V_a to V_b, the gas in a cylinder takes the following heat from the hot reservoir:

$$Q = \int_{V_a}^{V_b} \frac{T}{V} dV = T \ln \frac{V_b}{V_a} \, . \tag{3.2.4}$$

The value

$$H = \ln \frac{V_b}{V_a} \tag{3.2.5}$$

is the decrease of entropy as a result of receiving heat Q from the hot reservoir. The efficiencies of engines 1, 2, 3 are equal to x_1, x_2, x_1. We designate the initial volumes of gas in cylinders 1, 2, 3 as U_1, U_2, U_3, and the final ones as V_1, V_2, V_3, respectively. The work produced by the engines are

$$W_1 = x_1 T_1 \ln \frac{V_1}{U_1}, \tag{3.2.6}$$

$$W_2 = x_2 T_2 \ln \frac{V_2}{U_2}, \tag{3.2.7}$$

$$W_3 = x_1 T_3 \ln \frac{V_3}{U_3}. \tag{3.2.8}$$

The ratios of the initial and final volumes in the cylinders of engines 1 and 3 are related as

$$\left(\frac{V_1}{U_1}\right)^{T_1} = \left(\frac{V_3}{U_3}\right)^{T_2}, \tag{3.2.9}$$

where $T_1 > T_2$.

Now we begin our deduction of Stevens' law. In the descending reflexion, let the first engine cylinder be the perceptive input, and the third engine cylinder the cognizant output; $\frac{V_1}{U_1}$ is the intensity of the physical stimulus, and $\frac{V_3}{U_3}$ is the subject's cognizant evaluation of the intensity of the stimulus:

$$\frac{V_3}{U_3} = \left(\frac{V_1}{U_1}\right)^{\frac{T_1}{T_2}}. \tag{3.2.10}$$

In the ascending reflexion, the third engine cylinder is the perceptive input, and the first engine cylinder is the cognizant output. In this case, the cognizant evaluation of the intensity of the stimulus is

$$\frac{V_1}{U_1} = \left(\frac{V_3}{U_3}\right)^{\frac{T_2}{T_1}} . \tag{3.2.11}$$

The experimental subjects give evaluations proportional to $\frac{V_3}{U_3}$, in the descending reflexion, and to $\frac{V_1}{U_1}$ in the ascending reflexion.

Expressions (3.2.10) and (3.2.11) correspond to Stevens' law: in the first expression, the descending reflexion:

$$\frac{V_3}{U_3} = \psi_S, \quad \frac{V_1}{U_1} = S, \quad \frac{T_1}{T_2} = \beta ;$$

in the second expression, the ascending reflexion:

$$\frac{V_1}{U_1} = \psi_S, \quad \frac{V_3}{U_3} = S, \quad \frac{T_2}{T_1} = \beta .$$

For the descending reflexion, $\beta = \frac{T_1}{T_2} > 1$, and for the ascending $\beta = \frac{T_2}{T_1} < 1$. The exact value $\beta = 1$ cannot be realized in this model.

Let us deduce Weber-Fechner's law. Consider the problem of how the subject's perceptual evaluations relate to his feelings. We connect feelings with the work produced by engines. Using (3.2.10) we can write (3.2.8) as

$$W_3 = x_1 T_2 \ln \frac{V_1}{U_1} , \tag{3.2.12}$$

and (3.2.6) as

$$W_1 = x_1 T_2 \ln \frac{V_3}{U_3} . \tag{3.2.13}$$

Expression (3.2.12) corresponds to descending reflexion and (3.2.13) to ascending reflexion. The subjects give evaluations proportional to

W_3 and W_1. In this way we obtain logarithmic expressions relating the subject's feelings to the intensity of stimuli. These expressions correspond to the Weber-Fechner law (3.2.1). In (3.2.12)

$$W_3 = \psi_F, \quad V_1 = \varphi, \quad U_1 = \varphi_0,$$

and in (3.2.13)

$$W_1 = \psi_F, \quad V_3 = \varphi, \quad U_3 = \varphi_0 .$$

The multiplier $x_1 T_2$ may be considered a constant.

Stevens' power law corresponds to direct evaluations of the intensity of physical stimuli. Weber-Fechner's logarithmic law corresponds to the subjects' evaluations of the intensity of their feelings about stimuli. One support to this interpretation is the fact that all Tempered musical scales are logarithmic.

3.3. Generating musical intervals

For millennia, the nature of musical intervals has puzzled musicologists. Physicist Richard Feynman wrote (1966, 50-1):

> We may ask ourselves if we better than Pythagoras understand now why only some tunes are pleasant to our ear. The general theory of aesthetics is not advanced more in our time than it was during Pythagoras.

Hermann Helmholtz (1885/1954) named the set of "attractive" intervals the Scale of Harmonical. They were not deduced formally but rather chosen based on their sounding (see McClain, 1976, 1987; Burns & Ward, 1982):

$$\frac{1}{1}\,\frac{15}{16}\,\frac{9}{10}\,\frac{8}{9}\,\frac{5}{6}\,\frac{4}{5}\,\frac{3}{4}\,\frac{2}{3}\,\frac{5}{8}\,\frac{3}{5}\,\frac{4}{7}\,\frac{5}{9}\,\frac{8}{15}\,\frac{1}{2} \tag{3.3.1}$$

The above ratios (except the octave, $\frac{1}{2}$, and unison, $\frac{1}{1}$) can be written as fractions of the type

$$\frac{n+1}{n+2} \quad \text{or} \quad \frac{1}{2}\frac{n+2}{n+1}\,, \tag{3.3.2}$$

and $\frac{2}{3}$ and $\frac{3}{4}$ may be written as either of the two.

Let us construct a model of the subject generating musical intervals using the physical model of psychological processes. We will do this with the help of a three-engine unit and suppose that, in the descending reflexion, the ratio of work for engines 1 and 2 is an integer, and, in the ascending reflexion, the ratio of work for engines 3 and 2 is an integer (see Fig. 3.3.1).

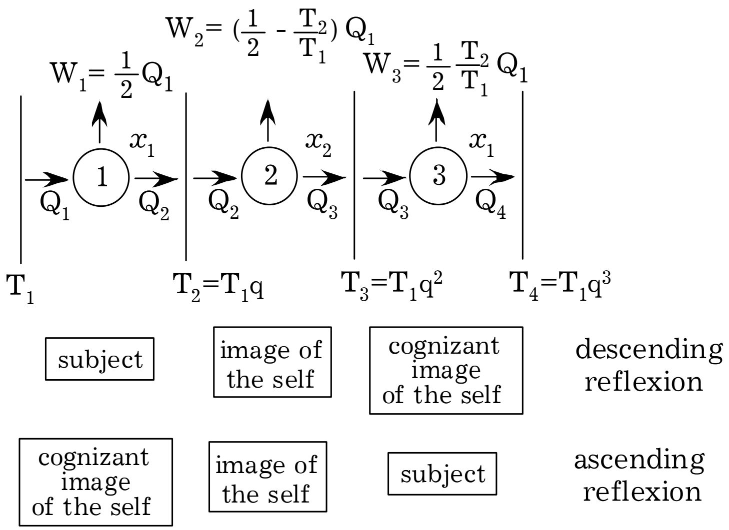

Fig. 3.3.1. Unit corresponding to a musician.
In the descending reflexion, the subject corresponds to the left engine, in the ascending to the right one

With such integer relations, the subject and the subject's image of the self are in resonance (we say this metaphorically, because in classical thermodynamics there is no concept of time). The value of X_1 corresponds to a musical interval, and W_1, W_2, W_3 correspond to the subject's feelings (emotions). In the descending reflexion, W_1 is the subject's emotions, W_2 is the subject's image of the self's emotions, and W_3 is the subject's cognizant emotions. In the ascending reflexion, W_3 is the subject's emotions, W_2 is the subject's image of the self's emotions, and W_1 is the subject's cognizant emotions. We assume that the pressure toward the positive and negative poles is the same, i.e., $x_1 = \frac{1}{2}$.

Deduction of mathematical form of musical intervals

In the descending reflexion, engines 1 and 2 are in resonance, and the following expression corresponds to the musician:

$$\frac{W_2}{W_1} = \frac{\left(\frac{1}{2} - \frac{T_2}{T_1}\right)}{\frac{1}{2}} = \frac{1}{k+1}, \tag{3.3.3}$$

where k a positive integer. We derive from (3.3.3) that

$$\frac{T_1 - T_2}{T_1} = \frac{k+2}{2k+2}, \tag{3.3.4}$$

and

$$X_1 = \frac{x_1}{\frac{T_1 - T_2}{T_1}} = \frac{\frac{1}{2}}{\frac{k+2}{2k+2}} = \frac{k+1}{k+2}. \tag{3.3.5}$$

In the ascending reflexion, engines 2 and 3 are in resonance, and the following expression corresponds to the musician:

$$\frac{W_2}{W_3} = \frac{\left(\frac{1}{2} - \frac{T_2}{T_1}\right)}{\frac{1}{2}\frac{T_2}{T_1}} = R, \tag{3.3.6}$$

where R is either k, or $1/k$. We derive from (3.3.6) that

$$\frac{T_1 - T_2}{T_1} = \frac{R+1}{R+2}. \tag{3.3.7}$$

Hence,

$$X_1 = \frac{\frac{1}{2}}{\frac{R+1}{R+2}} = \frac{1}{2}\frac{R+2}{R+1}. \tag{3.3.8}$$

With $R=k$ the musician generates intervals of the type

$$X_1 = \frac{1}{2}\frac{k+2}{k+1}, \tag{3.3.9}$$

and with $R=1/k$ he generates the intervals

$$X_1 = \frac{2k+1}{2k+2}. \tag{3.3.10}$$

It follows from (3.3.5), (3.3.9), and (3.3.10) that the musician generates intervals which can be written as

$$\frac{n+1}{n+2}, \quad \frac{1}{2}\frac{n+2}{n+1}. \tag{3.3.11}$$

We saw that the set of intervals separated by Helmholtz, except $\frac{1}{1}$ and $\frac{1}{2}$, consists of the intervals of the type (3.3.11). We will call intervals of this type *elite*.

Deduction of the set of musical intervals

There are countless fractions of the type (3.3.11). The set selected by Helmholtz consists of only 14 such intervals.

Interval d' is called an octave complement to interval d, if $dd' = \frac{1}{2}$. Most musical intervals in the set (3.3.1) have their octave complements there, except $\frac{8}{9}$ and $\frac{4}{7}$ whose octave complements are ($\frac{9}{16}$ and $\frac{7}{8}$). Let us add them to the Helmholtz set. In addition, we will also include consonance $\frac{6}{7}$ and its octave complement $\frac{7}{12}$ to (3.3.1) and obtain a new set which we name the *set of harmonical intervals*. All four added intervals are elite.

We will demonstrate now how these particular intervals are derived from the heat model. An interval is one of the simplest structural units in music. The next in complexity is a triad. It consists of three sounds which generate three intervals (Fig. 3.3.2).

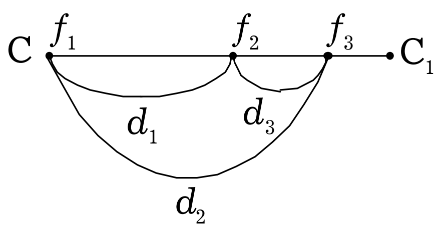

Fig. 3.3.2. Triad. Frequency f_1 corresponds to sound C; frequencies f_2 and f_3 correspond to two other sounds; d_1, d_2 and d_3 are the intervals between the sounds;

$$f_1 < f_2 < f_3 \leq 2f_1$$

The intervals are:

$$d_1 = \frac{f_1}{f_2}, \quad d_2 = \frac{f_1}{f_3}, \quad d_3 = \frac{f_2}{f_3}.$$

It is easy to see that

$$d_3 = \frac{d_2}{d_1}. \tag{3.3.12}$$

Let us consider a musician who can generate only elite intervals, i.e., intervals of the type (3.3.11). When d_1 and d_2 are elite, it may happen that d_3, calculated by (3.3.12), is not elite. Thus, to meet the condition that all three intervals be elite, only some sets of d_1, d_2, d_3 are solutions to equation (3.3.12). The elite intervals d_1 and d_2 will be called *compatible*, if the interval d_3 corresponding to them is also elite. The set of intervals compatible with an interval d^* will be called the interval d^* *suite*.

Statement 1. The set of harmonical intervals consists of unison ($\frac{1}{1}$), the intervals in the suite of the fifth ($\frac{2}{3}$), and their octave complements.

The fifth may be either interval d_2 or interval d_1.

If $d_2 = \frac{2}{3}$, then $d_3 = \dfrac{\frac{2}{3}}{d_1}$, which can be written as

$$\frac{k_3 + 1}{k_3 + 2} = \frac{\dfrac{2}{3}}{\dfrac{k_1 + 1}{k_1 + 2}}, \tag{3.3.13}$$

where k_1 and k_3 are positive integers.

If $d_1 = \frac{2}{3}$, then $d_3 = \dfrac{d_2}{\frac{2}{3}}$ or

$$\frac{k_3 + 1}{k_3 + 2} = \frac{\dfrac{1}{2}\,\dfrac{k_2 + 2}{k_2 + 1}}{\dfrac{2}{3}}. \tag{3.3.14}$$

Expressions (3.3.13) and (3.3.14) are Diophantine equations:

$$k_1 = \frac{k_3 + 5}{k_3 - 1}, \text{ where } k_3 \geq 2, \tag{3.3.15}$$

and

$$k_2 = \frac{2k_3 + 8}{k_3 - 2}, \text{ where } k_3 \geq 3. \tag{3.3.16}$$

Let us solve the Diophantine equation (3.3.15): if $m = k_3 - 1$, then

$$k_1 = 1 + \frac{6}{m}.$$

Since k_1 must be an integer, m may only be equal to 6, 3, 2, or 1. Whence,

$$k_1 = 2, 3, 4, 7.$$

Therefore, from all intervals of the type $\frac{k_1 + 1}{k_1 + 2}$, only the following four can be part of the suite of the fifth:

$$\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{8}{9} \tag{3.3.17}$$

The Diophantine equation (3.3.16) can be presented as

$$k_2 = 2 + \frac{12}{m}, \text{ where } m = k_3 - 2.$$

Since k_2 is an integer, m = 12, 6, 4, 3, 2, 1. Hence,

$$k_2 = 3, 4, 5, 6, 8, 14$$

and the suite of the fifth includes the intervals of the type $\frac{1}{2}\frac{k_2 + 2}{k_2 + 1}$:

$$\frac{5}{8}\frac{3}{5}\frac{7}{12}\frac{4}{7}\frac{5}{9}\frac{8}{15}. \tag{3.3.18}$$

By combining intervals (3.3.17) and (3.3.18) we obtain the suite of the fifth:

$$\frac{8}{9}\,\frac{5}{6}\,\frac{4}{5}\,\frac{3}{4}\,\frac{5}{8}\,\frac{3}{5}\,\frac{7}{12}\,\frac{4}{7}\,\frac{5}{9}\,\frac{8}{15}\,.$$

By adding to this set, unison ($\frac{1}{1}$) and the octave complements, which are not included in this obtained set, we obtain the final result:

$$\begin{matrix} \frac{1}{1} & \frac{8}{9} & \frac{5}{6} & \frac{4}{5} & \frac{3}{4} & \frac{5}{8} & \frac{3}{5} & \frac{7}{12} & \frac{4}{7} & \frac{5}{9} & \frac{8}{15} \\ \frac{1}{2} & \frac{9}{16} & & & \frac{2}{3} & & & \frac{6}{7} & \frac{7}{8} & \frac{9}{10} & \frac{15}{16} \end{matrix}\,. \qquad (3.3.19)$$

This set, as here derived, coincides with the set of attractive intervals.

So, we have proved that if the fifth ($\frac{2}{3}$) is used as a basic interval, a set of the corresponding intervals consists of all harmonical intervals. European music is based on the fifth. There are also countries where music is based on the fourth ($\frac{3}{4}$). For example, a developed musical system based on the fourth exists on the island of Java, it is called Pelog (Kunst, 1949). We can find a set of intervals based on the fourth in the same way as we found the set of intervals based on the fifth. It turned out that these sets coincide.

Statement 2. The set of harmonical intervals consists of the unison ($\frac{1}{1}$), the suite of the fourth ($\frac{3}{4}$) and their octave complements.

The following equations for the fourth correspond to (3.3.13) and (3.3.14) for the fifth:

$$\frac{k_3+1}{k_3+2}=\frac{\frac{3}{4}}{\frac{k_1+1}{k_1+2}}, \qquad (3.3.20)$$

$$\frac{k_3+1}{k_3+2}=\frac{\frac{1}{2}\frac{k_2+2}{k_2+1}}{\frac{3}{4}}. \tag{3.3.21}$$

They can be rewritten as Diophantine equations:

$$k_1 = 2 + \frac{12}{m}, \tag{3.3.22}$$

this implies m = 12, 6, 4, 3, 2, 1;

$$k_2 = 1 + \frac{6}{m}, \tag{3.3.23}$$

and m = 6, 3, 2, 1.

We met similar equations earlier but here sets of k_1 and k_2 values are reversed in comparison with the case with the fifth. Equations (3.3.22) and (3.3.23) allows us to find the fourth's suite:

$$\frac{15}{16}\ \frac{9}{10}\ \frac{7}{8}\ \frac{6}{7}\ \frac{5}{6}\ \frac{4}{5}\ \frac{2}{3}\ \frac{5}{8}\ \frac{3}{5}\ \frac{9}{16}. \tag{3.3.24}$$

Let us add here unison ($\frac{1}{1}$):

$$\frac{1}{1}\ \frac{15}{16}\ \frac{9}{10}\ \frac{7}{8}\ \frac{6}{7}\ \frac{5}{6}\ \frac{4}{5}\ \frac{2}{3}\ \frac{5}{8}\ \frac{3}{5}\ \frac{9}{16}. \tag{3.3.25}$$

To the received set, we add the octave complements, which are not included in it:

$$\frac{1}{1}\ \frac{15}{16}\ \frac{9}{10}\ \frac{7}{8}\ \frac{6}{7}\ \frac{5}{6}\ \frac{4}{5}\ \frac{2}{3}\ \frac{5}{8}\ \frac{3}{5}\ \frac{9}{16}$$

$$\frac{1}{2}\ \frac{8}{15}\ \frac{5}{9}\ \frac{4}{7}\ \frac{7}{12}\qquad \frac{3}{4}\qquad \frac{8}{9} \tag{3.3.26}$$

Thus, we have obtained the set (3.3.26) of harmonical intervals based on the fourth equivalent to the set based on the fifth (3.3.19).

Statement 3. The set of harmonical intervals consists of the unison ($\frac{1}{1}$), the octave ($\frac{1}{2}$), the suite of the fifth and the suite of the fourth.

Our deduction of the set of harmonical intervals was based on the two laws of thermodynamics. This makes us think that this set is a fundamental attribute of the nature and we may use it for selection of the Universe models, as we use the anthropic principle.