Chapter 13
Military decisions

In this chapter, we will show how the theory of reflexive games can be used for predicting decisions made on the battlefield. First, we will examine the theory by comparing its results with obvious intuitive predictions; then, we will model situations of military interaction.

13.1. Intuition and predictions by the model

A battalion commander has received an order to assemble troops near a river and, if possible, to force the river crossing. Three crossings exist in the area of the battalion's location: A, B, C. Each of them is defended by an enemy platoon, which constitutes a serious obstacle. What decision will the commander make? Will he cross the river at point A, point B, or point C, or will he not cross at all? To answer this question, we have only the brief description given above. So, we must look for other constraints on the commander's choice. We cannot find any, because there is not enough information. This means that the subject's choice depends on factors not included in the description, and we cannot predict his choice; i.e., from our position, the battalion commander has *freedom of choice*.

Suppose that we construct a model based on the given situation, and it predicts that the commander will choose to cross the river at point A. This prediction would undermine our trust in the model, because in the description there are no differences between A, B and C. We will consider a prediction satisfactory only if it states that the commander can choose any crossing, i.e., that he has freedom of choice.

Check now what our model predicts. The following graph corresponds to the situation (Fig.13.1.1):

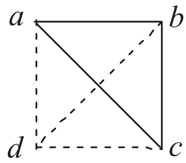

Fig. 13.1.1. Graph of relations

Node d depicts the battalion, nodes a, b and c depict platoons defending crossings A, B and C, respectively. The graph in Fig. 13.1.1 corresponds to the polynomial

$$d + abc \tag{13.1.1}$$

and to the diagonal form

$$\begin{array}{lll} & & [a]\,[b]\,[c] \\ & [d] + [abc] & \\ [d + abc] & & . \end{array} \tag{13.1.2}$$

The equation for the battalion commander is

$$d = d + abc. \tag{13.1.3}$$

The universal set consists of three actions: α, β, γ, where α is to cross the river at point A, β to cross at point B, and γ to cross at point C. Only $\{\alpha\}$, $\{\beta\}$, $\{\gamma\}$ and 0 are realizable alternatives. The choice of 0 means that the commander makes the decision not to force the river crossing. Each enemy's platoon influences the commander not to use the crossing that the platoon defends:

$$a = \overline{\{\alpha\}} : \text{do not use crossing A},$$
$$b = \overline{\{\beta\}} : \text{do not use crossing B},$$
$$c = \overline{\{\gamma\}} : \text{do not use crossing C},$$

or

$$a = \{\beta, \gamma\},$$
$$b = \{\alpha, \gamma\},$$
$$c = \{\alpha, \beta\}.$$

We substitute these values into (13.1.3) and obtain

$$d = d + \{\beta, \gamma\}\{\alpha, \gamma\}\{\alpha, \beta\}. \tag{13.1.4}$$

Since

$$\{\beta, \gamma\}\{\alpha, \gamma\}\{\alpha, \beta\} = 0,$$

$$d = d, \tag{13.1.5}$$

i.e., the battalion commander has freedom of choice. We see that our model's prediction coincides with the conclusion made after intuitive analysis of the situation.

Suppose that the enemy fortifies the defense of crossings B and C, but leaves A's defense as is. What decision will be made by the commander under this condition? It is clear that crossing at A becomes more attractive than B and C, but the potential use of B and C must remain.

Let us model this situation. First, we determine the influences. Platoon a, which defends crossing A, does not repel the battalion, but attracts it:

$$a = \{\alpha\}. \tag{13.1.6}$$

The influences of platoons b and c remain as they were previously:

$$b = \{\alpha, \gamma\}, \tag{13.1.7}$$

$$c = \{\alpha, \beta\}. \tag{13.1.8}$$

We substitute these values into (13.1.3) and obtain:

$$d = d + \{\alpha\}\{\alpha, \gamma\}\{\alpha, \beta\}. \tag{13.1.9}$$

Since

$$\{\alpha\}\{\alpha, \gamma\}\{\alpha, \beta\} = \{\alpha\}, \tag{13.1.10}$$

equation (13.1.9) becomes

$$d = d + \{\alpha\}, \tag{13.1.11}$$

or

$$d = d + \{\alpha\}\bar{d} . \tag{13.1.12}$$

The solutions of this equation are given by the inequalities

$$1 \supseteq d \supseteq \{\alpha\}. \tag{13.1.13}$$

Thus,

$$d = \{\alpha, \beta, \gamma\}, \{\alpha, \beta\}, \{\alpha, \gamma\}, \{\alpha\}. \tag{13.1.14}$$

The battalion commander can choose one of four alternatives, each of which contains action α. There is no other action that is included in all alternatives. So, crossing at A is *singled out*. With the choice of any alternative, the battalion commander can use this particular crossing.

We see that, in simple cases, our model's predictions correspond to those of intuition and common sense.

13.2. Choosing a path

Consider a more complicated case. The separate battalion d has the goal of descending from the mountains into the valley. The enemy's platoon, a, tries to hold the battalion in the mountains. All routes into the valley go through villages b and c. The villages' inhabitants are hostile to the battalion and support the enemy. Moreover, they are in conflict with each other. The situation corresponds to the graph in Fig.13.2.1.

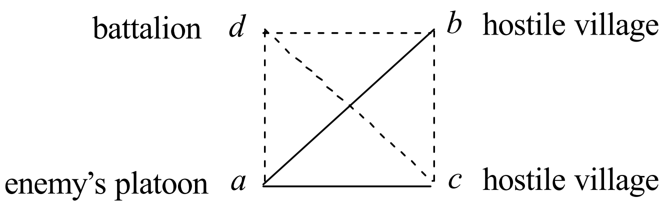

Fig. 13.2.1. Graph of relations

The universal set of actions for the commander contains all routes descending from the mountains into the valley. We assume that they are incompatible, i.e., the battalion can move along one route only. The set of alternatives M consists of all subsets of the universal set including the empty set, which corresponds to a refusal to make a descent. Set a consists of routes which the enemy's behavior inclines the battalion commander to use. For example, some routes are not covered by the enemy, which may incline the commander to use them. Sets b and c contain routes, which the villages' inhabitants incline the commander to use. The graph in Fig.13.2.1 is decomposable; it corresponds to the polynomial

$$d + a(b + c) \tag{13.2.1}$$

and the diagonal form

$$\begin{array}{llll} & & & [b] + [c] \\ & & [a]\,[b + c] & \\ & [d] + [a(b + c)] & & \\ [d + a(b + c)] & & & . \end{array} \tag{13.2.2}$$

The equation for the battalion commander is

$$d = d + a\ . \tag{13.2.3}$$

There are no variables b and c in this equation, so that, the commander's choice does not depend on the hostile villages' inhabitants. By solving (13.2.3), we find

$$1 \supseteq d \supseteq a\ . \tag{13.2.4}$$

When a is not empty, the battalion commander can choose any alternative containing a as a subset, and after that, he can realize one of the subsets of this alternative, which is a particular route. When $a = 0$, i.e., the enemy does not pressure the commander toward any route, the commander has freedom of choice and can choose any alternative, including the empty one. If the chosen alternative is not empty, the commander can single out a route for the battalion to descend to the valley.

What will be the commander's choice if the inhabitants of villages b and c are in cooperation instead of conflict? The graph of the relations in this case is given in Fig. 13.2.2.

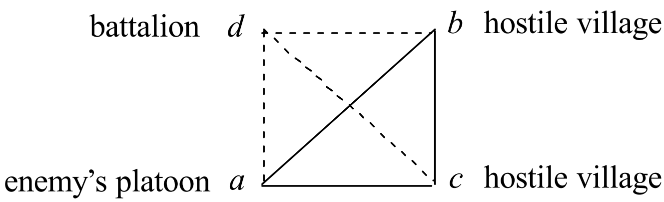

Fig. 13.2.2. Graph of relations

This graph corresponds to the polynomial

$$d + abc \tag{13.2.5}$$

and the diagonal form

$$\begin{array}{lll} & & [a\}\ [b]\ [c] \\ & [d] + [abc] & \\ [d + abc] & & \end{array} \quad . \tag{13.2.6}$$

The equation for the battalion commander is

$$d = d + abc, \tag{13.2.7}$$

hence,

$$1 \supseteq d \supseteq abc\ . \tag{13.2.8}$$

Thus, when the inhabitants of the hostile villages a and b cooperate with each other, the commander's choice depends on their influence.

For example, village b inclines the battalion commander to choose alternative 0 (b=0). Pressure toward zero means that subject b, by his behavior, prompts the battalion to inaction. As a result, the battalion commander obtains freedom of choice.

13.3. Reflexive control

Consider the scenario corresponding to Fig.13.2.1. The commander's choice is given by the inequalities $1 \supseteq d \supseteq a$. Let the head of the enemy platoon decide to ambush the battalion on one of the routes and to use reflexive control as a tool. If he realizes influence $a = 1$ and persuades the battalion commander that every route to the valley is safe, he will not be able to predict the battalion's route and will not be able to set the ambush. If the head of the enemy unit realizes influence $a = 0$, i.e., all routes down are dangerous, then the battalion commander will obtain freedom of choice, and it will again be impossible to predict his route. The best expedient is to pressure the commander toward choosing a concrete route λ, i.e., to use influence $a = \{\lambda\}$. Then the battalion commander's choice will be given by the inequalities

$$1 \supseteq d \supseteq \{\lambda\}, \tag{13.3.1}$$

which means that the choice will be made from a set of subsets, each of which contains λ. With this reflexive control, route λ will be included in each alternative. There is no guaranty, however, that the battalion will use this route.