Chapter 4
Initial model

In this chapter we introduce the basic model that will be used in subsequent analysis. Additional extensions of the model are introduced in Chapter 6.

4.1. A general schema

We assume that a *subject* is an individual or an organization that includes people. Several interacting subjects constitute a group. Every two subjects in a group are either in a relation of cooperation or in one of conflict. Within the framework of our model, the concepts of cooperation and conflict are fundamental and cannot be reduced to other concepts. There is a set of actions that subjects can perform. In the basic model, every subject in the group is able to perform each of those actions.

A subset of actions may or may not be capable of being performed simultaneously. A set of actions is called an *alternative*. A subject chooses an alternative and then realizes any compatible set of actions from the chosen alternative. A subject's choice depends on the relationships among group members and the influences of other subjects. In addition, the subject has an *intention* to choose one or another of the alternatives. The intention is considered as *self-influence*. Subjects are either intentional or not intentional. The former have only intentions that can become reality; the latter have arbitrary intentions.

A subject is capable of integrating the influences of all subjects, including self-influence, into the influence of the group as a whole. For every subject, there is a corresponding diagonal form based on the graph of relationships within the group. The diagonal form describes the subject's reflexion, i.e., the hierarchy of the subject's images of the self, and at the same time represents the function of the subject's choice. A formal procedure for computing the value of this function is regarded as a model of mental choice generation. Relations within the group and influences of other members impose constraints on the subject's choice. The model allows us to predict the subject's possible choices with account taken of these restrictions.

4.2. Representation of the subject

We assume that the subject can perform actions α_1, α_2, . . . , α_S, $S \geq 1$. All of these actions are *acceptable* for the subject, i.e., the subject is able to perform them both technically and morally. *The relation of preference on the set of actions is not given.* The set of actions is regarded as the universal set. Set M of all subsets of the universal set, including the empty set, is the set of alternatives. In other words, each alternative is a subset of the universal set of actions. Choice of the empty set is interpreted as the subject's refusal to choose any non-empty alternative. The subject's activity consists of choosing an alternative from set M, and then realizing the choice. Therefore, the model distinguishes between 'choice' and 'realization of choice'. On the set of all subsets of actions, there is given a unary relation of realization. The empty set and a one-element set can always be realized. For other sets the possibility or impossibility of realization must be specially given. If a non-empty alternative is chosen, then any of its non-empty realizable subsets can be realized (but only one). To clarify the distinction between 'choice' and 'realization', consider an example. Let the universal set consist of two actions:

α_1 – turn left
α_2 – turn right

The Boolean lattice of alternatives appears as follows:

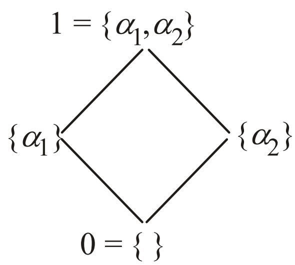

Fig. 4.2.1. Boolean lattice of the set of alternatives

The choice of { } means that the subject chooses inactivity; he will perform neither α_1 nor α_2. The choice of $\{\alpha_1\}$ means that the subject will perform only action α_1, and the choice of $\{\alpha_2\}$ that he will perform only action α_2. Consider the alternative $\{\alpha_1, \alpha_2\}$. Since the subject cannot perform actions, α_1 (turn left) and α_2 (turn right) at the same time, such actions are *incompatible*, and alternative $\{\alpha_1,\alpha_2\}$ *is not realizable*. After choosing it, however, the subject can realize either subset $\{\alpha_1\}$, or subset $\{\alpha_2\}$. The meaning of choice $\{\alpha_1,\alpha_2\}$ is rejection of the empty alternative { }. This is the reason for assigning a unary relation of realization on set M. There are situations in which actions α_1, and α_2 are compatible. For example, α_1 is buying a watch and α_2 is buying a telephone. A person can buy both a watch and a telephone at the same time. In this case, after choosing $\{\alpha_1, \alpha_2\}$, the subject can realize any one of the three subsets of actions: $\{\alpha_1\}$, $\{\alpha_2\}$, $\{\alpha_1,\alpha_2\}$.

A diagonal form corresponds to a particular subject and defines the subject's choice function. Consider subject a_k:

$$\Phi_k = \Phi(a_1, \ldots, a_k, \ldots, a_n). \qquad (4.2.1)$$

Variables $a_1, \ldots, a_k, \ldots, a_n$ are defined on set M of all subsets of the universal set of actions. The value of function Φ_k is subject a_k's choice. Variable a_i corresponds to subject a_i. The value of a_i is the alternative that a_i inclines a_k to choose. The value of a_k is subject a_k's intention to choose a certain alternative.

As we mentioned earlier, a subject can be intentional or not intentional. The latter can have any intention, i.e., variable a_k can take on any value from M; the subject's choice is given by function (4.2.1). An intentional subject with given values of $a_1, \ldots, a_{k-1}, a_{k+1} \ldots, a_n$, has only intentions that can become a choice. Each intention is a solution of the equation (4.2.2):

$$a_k = \Phi_k\,(\,a_1, \ldots, a_k, \ldots,\ a_n). \tag{4.2.2}$$

The solution is also interpreted as a possible choice. Equation (4.2.2) may have no solution. This is interpreted as the subject's inability to make an intentional choice.

Consider now the connection between diagonal form and the mental aspect of the subject's activity. A diagonal form is constructed by decomposition of the graph of a group containing the subject. The diagonal form represents the subject with a hierarchy of images of the self. It is a tree at whose ramifications there are polynomials in brackets. These polynomials constitute a partially-ordered set; every polynomial located above and to the right of another polynomial is considered to 'follow' the latter. Each polynomial in the diagonal form has its own diagonal form for which it is the bottom-most element. We say that diagonal form B follows diagonal form A, if B's bottom-most polynomial follows A's bottom-most polynomial. In considering diagonal forms as depictions of the subject, we say that B is A's image of the self, if B follows A. This allows us to construct recursive statements of the type "A_m is A_{m-1}'s image of the self that is A_{m-2}' image of the self that is ... A_1's image of the self." Each statement corresponds to the path along the branches from an end to the root. Thus, the chain of statements is finite. Each diagonal form has a unique set of such chains.

As an example, consider diagonal form (4.2.3):

$$\begin{array}{lll} & & [a_2]\,[a_3] \\ & [a_1] + [a_2\, a_3] & \\ [a_1 + a_2\, a_3] & & \end{array} \quad . \tag{4.2.3}$$

A partial order of polynomials constituting this diagonal form is given in Fig.4.2.2:

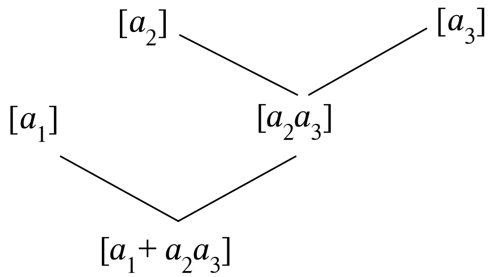

Fig. 4.2.2. Partial order of polynomials

A partially-ordered set of diagonal forms is given in Fig. 4.2.3:

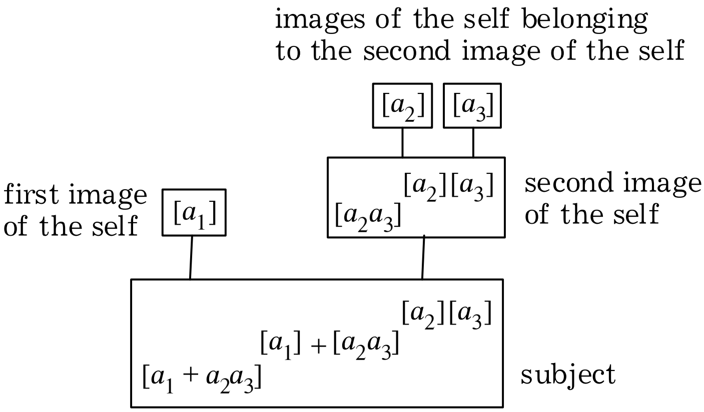

Fig. 4.2.3. Partial order of diagonal forms

Each lower polynomial corresponds to a group influencing the subject. Symbol · means cooperation, and + confrontation. The value of the polynomial is interpreted as the influence of a group as a whole. The polynomials following the bottom-most one correspond to minimal strata into which the initial group is divided. Each minimal stratum influences the subject's image of the self corresponding to it. Relations between images are predetermined by the relations between minimal strata influencing them. If minimal strata are in a state of cooperation, then the images are in a state of cooperation; if minimal strata are in conflict, then the images are in conflict.

A subject consisting of one letter is called *elementary*. Such a subject corresponds to polynomial $[a]$; this is also the subject's diagonal form.

A non-elementary subject is depicted by a diagonal form of the type

$$\Phi = P^{W}, \tag{4.2.4}$$

where P is the bottom-most polynomial of the diagonal form; $W = A_1 * A_2 * \ldots * A_k$; $k \geq 2$; $*$ either $\cdot$, or $+$, and A_i diagonal forms representing the subject's images of the self.

Expression W is the subject's *integral image of the self* that, in our model, consists of a collection of the images of self in cooperation or conflict with one another. The integral image of the self for subject (4.2.3) is

$$W = [a_1] + [a_2\ a_3]^{[a_2]\,[a_3]}, \tag{4.2.5}$$

where $A_1 = [a_1]$ is the first image of the self; $A_2 = [a_2\ a_3]^{[a_2]\,[a_3]}$ the second image of the self; + means that the images are in conflict.

Expression (4.2.4) gives a function that we call the *reflexion function*. It can be represented as

$$\Phi = P + \overline{W}. \tag{4.2.6}$$

Variables P and W take on values from the set of alternatives. The value of P is interpreted as the alternative to which the group inclines the subject. Actions in set P are called *attractive to the group*, and those in $\overline{P}$ *unattractive to the group*. Let us emphasize that P is the set of actions attractive for the group, whose performance is *expected from a particular subject*. For another subject, the group's influence may be different and the set of expected actions may be different as well. Set W is the result of a choice made by the integral image of the self, that is, the result of the subject's *mental* choice. Since a mental choice of something means preference of this 'something', we say that W consists of actions *attractive for the subject*. Actions in $\overline{W}$ we will call *unattractive for the subject*. The value of Φ corresponds to the subject's choice.

Let us substantiate now our choice of function (4.2.6) as the reflexion function. We will demonstrate that, by means of this function, we insert *the anti-selfishness principle* into the model:

> *While pursuing his own personal goals, the subject may not cause harm to the group he is a member of.*

Within our framework of, this principle can be formulated as follows: it is unacceptable to choose an alternative with actions attractive to the subject but unattractive to the group.

Consider the Venn diagram (Fig.4.2.4).

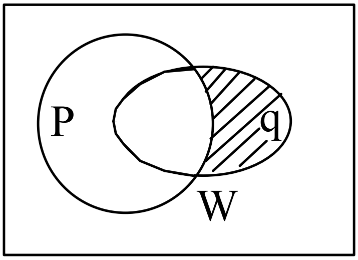

Fig. 4.2.4. Hatched set q is the set of prohibited actions

The set of prohibited actions is as follows:

$$q = \overline{P}W, \tag{4.2.7}$$

where $\overline{P}$ is the set of actions unattractive for the group, and W is the set of actions attractive for the subject.

The set of 'permitted' actions is

$$\overline{q} = P + \overline{W} . \qquad (4.2.8)$$

The subject chooses the set of all permitted actions, and, as a result, we obtain the function (4.2.6).

The choices of the subject and of every non-elementary subject in his hierarchy of images are based on the anti- selfishness principle.

Note that an action unattractive to the group may be included in the subject's choice if it is also unattractive to the subject, i.e., if it belongs to the set

$$\overline{P}\,\overline{W} = \overline{P}(P + \overline{W}) . \qquad (4.2.9)$$

Thus, *the subject may act against the group's interests, if by doing so the subject does not pursue his individual goals and is willing to sacrifice them.*

We have supposed that, in the subject's mental domain, a graph is gradually decomposed to minimal strata and single influences are integrated into the unified influence of the group, represented by the value of the corresponding polynomial. Let set A be the influence of one group, from the subject's point of view, and set B be the influence of another group. (A group may consist of a single individual.) We assume that if these groups are in cooperation, then, from the subject's point of view, they can come to a consensus, so that their joint influence inclines the subject toward choosing actions common to the interests of both groups. Therefore, the influence of groups that are in cooperation corresponds to the *intersection* of sets A and B:

$$AB.$$

If the groups are in conflict, there is no consensus; each group influences the subject independently. Therefore, the influence of groups that are in conflict corresponds to the *union* of the sets A and B :

$$A + B.$$

Similar considerations can be expressed regarding self-images. If images are in cooperation with each other, they are in a consensus; their joint choice is the intersection of their individual choices. If the images are in confrontation, a consensus is impossible; their joint choice is the union of the individual choices.

This substantiates the designation of cooperation by $\cdot$, and conflict by $+$.

4.3. Representation of a group

1. Let us consider a group consisting of subjects $a_1, a_2, \ldots, a_n$, где $n \geq 1$.

2. In any non-elementary group, we define the binary relations $\cdot$ (cooperation) and $+$ (conflict) (one of them may be empty). As a result, we obtain the relation graph G. In the framework of the *initial* model, we assume that graph G is decomposable.

3. A set of actions $\{\alpha_1, \alpha_2, ..., \alpha_S\}$, $S \geq 1$, is defined as common to all subjects. This is the universal set 1. A set of all subsets of the universal set (including the empty set) is interpreted as a set of alternatives and designated M.

4. The unary relation of realization is defined on set M for each subject.

5. A matrix of influence is constructed:
$\|p_{ij}\|$, i =1, ..., n; j=1, ..., n; where $p_{ij} \in M$; p_{ij} is the alternative, which subject a_i inclines subject a_j to choose. An element of the type p_{kk} is subject a_k's self-influence (intention).

6. Using the relation graph, we construct the diagonal form Φ, representing the hierarchy of images of the self and, at the same time, the choice function of each subject in the group.

7. Letters a_1, a_2, ..., a_n are variables. The same function corresponds to each subject:

$$\Phi = \Phi(a_1, a_2, \dots, a_n)\ . \tag{4.3.1}$$

8. If subjects are non-intentional, the value of variables for each of them are predetermined by the matrix $\|p_{ij}\|$, all elements of which are known. Element p_{kk} is interpreted as the subject's intention and, at the same time, as his self-influence. A matrix column number j contains influences on subject a_j. Thus, subject a_j corresponds to expression

$$\Phi_j = \Phi\ (p_{1j}, p_{2j}, \dots, p_{nj})\ . \tag{4.3.2}$$

Φ_j is an element of M; we interpret it as the alternative chosen by subject a_j. If Φ_j is the empty set, the subject realizes it, i.e., does not perform any actions. If Φ_j is not empty, then the subject realizes any of its non-empty realizable subset, and only one.

9. If subjects are intentional, the diagonal elements, p_{kk}, of their influence matrix $\|p_{ij}\|$ are not known in advance. These elements correspond to the intentions of the subjects constituting a group. They can be found from the following equations for p_{kk}:

$$p_{kk} = \Phi\ (p_{1k}, p_{2k}, \dots, p_{kk}, \dots, p_{nk})\ , \tag{4.3.3}$$

where $k = 1, 2, \dots, n$. The value of p_{kk} is interpreted as subject a_k's intention, his self-influence, and his choice, at the same time.

Equation of the type (4.3.3) may or may not have solutions. The absence of solutions means that, with a given relation graph and combination of influences, subject a_k cannot make an intentional choice. In this case, we say that subject a_k is *frustrated* or *in a state of frustration*. It is possible that any element of M is a solution of the equation (4.3.3). Then we say that subject a_k has *freedom of choice* or is *in the state of free choice*. We will say that the subject who can choose only the empty alternative { } is *in a* *passive state*, and the one who can choose a non-empty alternative is *in an active state*. Sometimes it is convenient to speak of the subject's choice, sometimes of the subject's state, especially in cases where the subject chooses between alternatives 1 and 0.

4.4. Examples for analysis

A group consists of three subjects: a_1, a_2, a_3. Two binary relations, $\cdot$ (cooperation) and + (conflict), are defined. Let subjects (a_1, a_3) and subjects (a_2, a_3) be connected by cooperation, and subjects (a_1, a_2) by conflict. Construct a graph corresponding to this group (Fig.4.4.1):

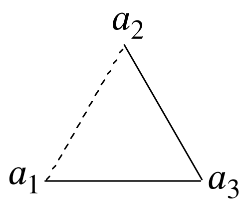

Fig. 4.4.1. Solid lines designate cooperation, and broken lines conflict

Let each subject be able to perform actions α_1, α_2, α_3. The set of actions $\{\alpha_1,\alpha_2,\alpha_3\}$ is the universal set 1. The set of all subsets of the set of actions is M. It is the set of alternatives. A unary relation of realization is given on it. Suppose that this is the same for all subjects: actions α_1 and α_3 cannot be performed at the same time. Thus,

$1 = \{\alpha_1, \alpha_2, \alpha_3\}$ is not realizable
$\{\alpha_1, \alpha_2\}$ is realizable
$\{\alpha_1, \alpha_3\}$ is not realizable
$\{\alpha_2, \alpha_3\}$ is realizable
$\{\alpha_1\}$ is realizable
$\{\alpha_2\}$ is realizable
$\{\alpha_3\}$ is realizable
$0 = \{\ \}$ is realizable

First consider subjects who are non-intentional and construct their own influence matrix. Let subject a_1 have the intention to choose alternative $\{\alpha_2, \alpha_3\}$ and incline subject a_2 to choose 0, and subject a_3 to choose $\{\alpha_2, \alpha_3\}$. Subject a_2 has the intention to choose $\{\ \}= 0$ and inclines a_1 to choose $\{\alpha_1, \alpha_3\}$ and a_3 to choose $\{\alpha_2\}$. Subject a_3 has the intention to choose 1 and inclines a_1 to choose $\{\alpha_2\}$ and a_2 to choose $\{\alpha_1, \alpha_2\}$. Column a_1 contains influences on subject a_1 from the self and from subjects a_2, a_3; column a_2 contains influences on a_2 from the self and a_1, a_3; column a_3 contains influences on a_3 from the self and a_1, a_2.

Table 4.4.1

Matrix of influences in a group of three non-intentional subjects. Lines show influences that subjects exert on others and the self; columns contain influences that are exerted on the subject.

	a_1	a_2	a_3
a_1	$a_1 = \{\alpha_2, \alpha_3\}$	$a_1 = 0$	$a_1 = \{\alpha_2, \alpha_3\}$
a_2	$a_2 = \{\alpha_1, \alpha_3\}$	$a_2 = 0$	$a_2 = \{\alpha_2\}$
a_3	$a_3 = \{\alpha_2\}$	$a_3 = \{\alpha_1, \alpha_2\}$	$a_3 = 1$

The polynomial corresponding to graph in Fig. 4.4.1 is

$$a_3(a_1 + a_2). \qquad (4.4.1)$$

Using the procedure of constructing a diagonal form when a polynomial is known, we obtain (4.4.2):

$$\begin{array}{lll} & & [a_1] + [a_2] \\ & [a_3]\,[a_1 + a_2] & \\ X = [a_3\,(a_1 + a_2)] & & \end{array} \quad . \qquad (4.4.2)$$

On the one hand, this form describes the hierarchy of images of the self, the same for all three subjects; on the other, it represents a function of choice, also the same for all three subjects. In the latter case, the diagonal form is regarded at as an exponential formula. It can be simplified:

$$X = a_1 + a_2 + \overline{a}_3 \,. \tag{4.4.3}$$

The alternatives chosen by subjects a_1, a_2, a_3 will be designated as $X_{a_1}, X_{a_2}, X_{a_3}$, respectively. By substituting values from the first, second, and third columns of the influence matrix into equation (4.4.3), we obtain

$$X_{a_1} = \{\alpha_2, \alpha_3\} + \{\alpha_1, \alpha_3\} + \overline{\{\alpha_2\}} = \{\alpha_1, \alpha_2, \alpha_3\} = 1 \,,$$

$$X_{a_2} = 0 + 0 + \overline{\{\alpha_1, \alpha_2\}} = \{\alpha_3\} \,,$$

$$X_{a_3} = \{\alpha_2, \alpha_3\} + \{\alpha_2\} + \overline{1} = \{\alpha_2, \alpha_3\} \,.$$

Subject a_1 chooses set $1 = \{\alpha_1, \alpha_2, \alpha_3\}$ and can realize any non-empty subset of this set of actions except $\{\alpha_1, \alpha_2, \alpha_3\}$ and $\{\alpha_1, \alpha_3\}$, which are non-realizable. Subject a_2 chooses set $\{\alpha_3\}$ consisting of one action α_3. A set of one action is always realizable. Subject a_3 chooses set $\{\alpha_2, \alpha_3\}$, corresponding to three realizable subsets: $\{\alpha_2, \alpha_3\}$, $\{\alpha_2\}$ and $\{\alpha_3\}$. Subject a_3 can realize any of these. Note that the model does not predict which of the realizable subsets will be realized.

Consider now intentional subjects. In this case, the subjects' intentions are not given in advance. They must be found by solving equations. The influence matrix is as follows:

Table 4.4.2
Matrix of influences in a group of three

	a_1	a_2	a_3
a_1	a_1	$a_1 = 0$	$a_1 = \{\alpha_2, \alpha_3\}$
a_2	$a_2 = \{\alpha_1, \alpha_3\}$	a_2	$a_2 = \{\alpha_2\}$
a_3	$a_3 = \{\alpha_2\}$	$a_3 = \{\alpha_1, \alpha_2\}$	a_3

When subjects are intentional, the diagonal elements, intentions a_1, a_2, a_3, are unknown values, unlike the defined sets in the case of non-intentional subjects. The following equations correspond to subjects a_1, a_2, a_3:

$$a_1 = a_1 + a_2 + \bar{a}_3\,, \tag{4.4.4}$$

$$a_2 = a_1 + a_2 + \bar{a}_3\,, \tag{4.4.5}$$

$$a_3 = a_1 + a_2 + \bar{a}_3\,. \tag{4.4.6}$$

These equations result from the successive substitutions of unknown a_1, a_2, a_3 for X in (4.4.3). Transform them to form $a_i = Aa_i + B\bar{a}_i$, $(i = 1, 2, 3)$:

$$a_1 = a_1 + (a_2 + \bar{a}_3)\bar{a}_1\,, \tag{4.4.7}$$

$$a_2 = a_2 + (a_1 + \bar{a}_3)\bar{a}_2\,, \tag{4.4.8}$$

$$a_3 = (a_1 + a_2)a_3 + \bar{a}_3\,. \tag{4.4.9}$$

In the first equation, the unknown is a_1. The values for a_2 and a_3 are taken from the a_1-column of the influence matrix. In the second equation, the unknown is a_2. The values for a_1 and a_3 are taken from the a_2-column. In the third equation, a_3 is unknown. The values of a_1 and a_2 are taken from the a_3-column.

The equation for subject a_1 is

$$a_1 = a_1 + \{\alpha_1, \alpha_3\}\bar{a}_1\,, \tag{4.4.10}$$

$A = 1$, $B = \{\alpha_1,\alpha_3\}$; $A \supset B$, thus, (4.4.10) is solvable. Its solutions satisfy the inequalities

$$1 \supseteq a_1 \supseteq \{\alpha_1, \alpha_3\}\,.$$

This implies the existence of two solutions, each of which contains the elements α_1 and α_3:

$$a_1 = \{\alpha_1,\ \alpha_2,\ \alpha_3\} = 1,$$
$$a_1 = \{\alpha_1,\ \alpha_3\}.$$

Subject a_1 can choose any of these solutions and realize any non-empty realizable subset of the chosen set of actions.

The equation for subject a_2 is

$$a_2 = a_2 + \{\alpha_3\}\overline{a}_2 , \tag{4.4.11}$$

$A = 1$, $B = \{\alpha_3\}$, $A \supset B$, thus, (4.4.11) has solutions satisfying the inequalities

$$1 \supseteq a_2 \supseteq \{\alpha_3\} .$$

There are four solutions corresponding to the above inequalities, and each of them contains the element α_3.

$$a_2 = \{\alpha_1, \alpha_2, \alpha_3\} = 1,$$
$$a_2 = \{\alpha_1, \alpha_3\},$$
$$a_2 = \{\alpha_2, \alpha_3\},$$
$$a_2 = \{\alpha_3\}.$$

The subject can choose any of the four above sets and then realize any realizable subset of the chosen set.

The equation for the subject a_3 is

$$a_3 = \{\alpha_2, \alpha_3\}a_3 + \overline{a}_3 . \tag{4.4.12}$$

$A = \{\alpha_2, \alpha_3\}$, $B = 1$; $A \subset B$, which implies that equation (4.4.12) does not have a solution. Subject a_3 cannot make an intentional choice, and we conclude that he is in a state of frustration.

4.5. The anti-selfishness principle and mathematical formalism

From the formal point of view, the anti-selfishness principle is represented by the following equation:

$$\Phi = P + \overline{W} ,$$

where Φ is the subject's choice. We will consider below the mathematical formalism corresponding to the mental process of generating P and W.

A non-intentional subject a_k is represented as

$$\Phi(a_1, a_2, \ldots, a_k, \ldots a_n) = P + \overline{W}\ , \tag{4.5.1}$$

where P, W are functions

$$P = P\ (a_1, a_2, \ldots, a_k, \ldots, a_n), \tag{4.5.2}$$

$$W = W\ (a_1, a_2, \ldots, a_k, \ldots, a_n); \tag{4.5.3}$$

P is the set of actions attractive to the group for subject a_k, and W is the set of actions attractive for subject a_k (see (4.2.6)). By giving the values of influences on subject a_k from other subjects and from the self, we predetermine sets P and W.

For a intentional subject, sets P and W exist only for such collections of variable values a_1, a_2, ..., a_k, ..., a_n, for which the following equation holds:

$$a_k = \Phi(a_1, a_2, \ldots, a_k, \ldots, a_n). \tag{4.5.4}$$

In this case P and W can be considered as functions defined on the set of such collections. We see that the formalism itself generates sets P and W; they are not introduced into the model from without. We do not need to know in advance which actions are attractive to the group and which to the subject.

With fixed values of

$$a_1, a_2, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n\ ,$$

the values of P and W depend on choosing a_k's value that satisfies equation (4.5.4). Thus, the group's preference depends on the subject's choice. This is possible, because it is not a real group, but an image of the group formed in the subject's inner domain. The subject's choice influences his image of the group's preferences. In this way, the subject forecasts actions that would be attractive to the group subsequently to his choice. The subject's own preferences may also depend on his own choice.

To demonstrate how the anti-selfishness principle works in the formal model, we consider a intentional subject a_1 with diagonal form (4.4.2) and equation (4.4.7). Its solutions belong to the interval

$$1 \supseteq a_1 \supseteq (a_2 + \overline{a}_3) \ . \tag{4.5.5}$$

With values of a_1, a_2 and a_3 satisfying (4.5.5), the set of actions attractive to the group is

$$P = [a_3 \ (a_1 + a_2)], \tag{4.5.6}$$

and the set of actions attractive to the subject is

$$W = [a_3] \ [a_1 + a_2]^{[a_1] + [a_2]} = a_3. \tag{4.5.7}$$

Let the universal set of subject a_1 be $1 = \{\alpha, \beta, \gamma, \delta\}$, and let $a_2 = \{\alpha, \beta\}$ and $a_3 = \{\alpha, \beta, \gamma\}$. Suppose a_1 chooses the alternative $\{\alpha, \beta, \delta\} = a_2 + \overline{a}_3$, from the interval

$$1 \supseteq a_1 \supseteq (a_2 + \overline{a}_3).$$

In this case

$$a_1 = \{\alpha, \beta, \delta\},$$
$$P = \{\alpha, \beta\},$$
$$W = \{\alpha, \beta, \gamma\}.$$

The set of actions prohibited for the subject is

$$\overline{P} W = \{\gamma\} \ .$$

The set of actions unattractive to the subject is

$$\overline{W} = \{\delta\} \ .$$

The set of actions unattractive both to the subject and to the group is

$$\overline{P}\,\overline{W} = \{\delta\}\,.$$

The set of actions attractive to the subject and to the group is

$$PW = \{\alpha,\ \beta\}.$$

We see that set $\{\alpha, \beta, \delta\}$, chosen by the subject, consists of two actions, α and β, attractive both to the subject and to the group, plus one action, δ, unattractive to both the subject and the group. This choice is in accordance with the anti-selfishness principle, since one of the actions included in the choice that is unattractive to the group is also unattractive to the subject.

4.6. Actions that are always chosen and actions that are never chosen.

Let subject a correspond to the equation

$$a = Aa + B\overline{a}\ , \qquad (4.6.1)$$

where $A \supseteq B$. The solutions of this equation belong to the interval

$$A \supseteq a \supseteq B\ . \qquad (4.6.2)$$

If $A = B$, there is only one solution. If $A \supset B$, there are several solutions. In this case, the model does not allow us to predict which one of them will be chosen by the subject. But the model helps us to single out two special sets: the set of actions that are present in any choice, and the set of actions that are absent from all choices. We designate them as R and S, respectively.

The following correlations hold for any solution a satisfying (4.6.2):

$$Ra = R, \qquad (4.6.3)$$

$$Sa = 0. \tag{4.6.4}$$

Let us prove first that $R = B$. Any action from B is included in every choice by a belonging to the interval (4.6.2); not a single action from $\overline{B}$ possesses this quality. Thus, B is the set of actions that are present in any choice.

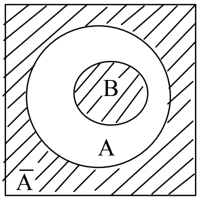

Fig. 4.6.1. Venn diagram. The hatched area in the center (B) is set R; the hatched area outside the larger circle $(\overline{A})$ is set S

We will prove now that $S = \overline{A}$. Not a single action from $\overline{A}$ is included in any choice by the subject, and any action from set A is included at least in one choice. Thus, $\overline{A}$ is the set of actions that are absent from all choices. The Venn diagram clarifies these considerations (Fig. 4.6.1).

Consider an example. Let subject a correspond to the equation

$$a = (b + c)a + c\overline{a} , \tag{4.6.5}$$

the universal set is $\{\alpha, \beta, \gamma, \delta \}$, b = $\{\alpha, \beta\}$ и c = $\{\delta\}$. For these values, we obtain

$$A = \{\alpha, \beta, \delta\}, B = \{\delta\}. \tag{4.6.6}$$

Solutions for equation (4.6.5) belong to the interval

$$\{\alpha, \beta, \delta\} \supseteq a \supseteq \{\delta\}. \tag{4.6.7}$$

It follows from (4.6.7) that equation (4.6.5) has four solution, i.e., the subject has four choices:

$$\{\alpha, \beta, \delta\}, \{\alpha, \delta \}, \{\beta, \delta \}, \{\delta\}.$$

The set of actions always chosen is

$$R = B = \{\delta\},$$

consisting of one action δ. The set of actions never chosen is

$$S = \overline{A} = \overline{\{\alpha, \beta, \delta\}} = \{\gamma\}\ ,$$

consisting of one action γ.

In the following, we will consider only intentional subjects.