Chapter 5
Theorems on variety

Let the universal set consist of one action α_1, (i.e., the universal set is $\{\alpha_1\}$); the set of its subsets, M, consists then of two sets: $1=\{\alpha_1\}$ and $0=\{\ \}$. If, under given conditions, the subject can choose only set 1, he is in a state of *activity*; if the subject can choose only set 0, he is in a state of *passivity*; if the subject can choose either 1, or 0, he is in a state of *free choice*; finally, if the subject cannot make a choice, he is in a state of *frustration*. Intuition suggests that there must exist groups, containing subjects in all four states, and groups in which there is a subject capable of being in each of the states, depending on the influences exerted by other subjects. We call these intuitive considerations the *minimal requirements for variety*, which must be satisfied by the model.

5.1. First theorem on variety

Formulation: There is at least one group, with a decomposable relation graph and an influence matrix, such that the group contains subjects in all four states.

Proof. Consider a group of subjects with relation table 5.1.1 and influence table 5.1.2:

Table 5.1.1

Table of relations

	a	*b*	*c*	*d*	*e*
a		·	+	·	·
b	·		+	·	·
c	+	+		·	·
d	·	·	·		+
e	·	·	·	+	

Table 5.1.2
Table of influences

	a	b	c	d	e
a	a	0	1	0	0
b	1	b	1	0	0
c	0	0	c	1	1
d	0	0	0	d	1
e	0	0	0	0	e

We construct a graph corresponding to the relation table:

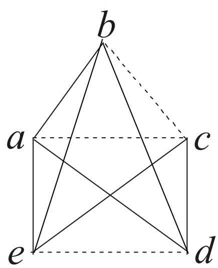

Fig. 5.1.1. Graph of relations

This graph is decomposable. Its polynomial is

$$(ab + c)\,(e + d) \tag{5.1.1}$$

and the diagonal form is

$$\begin{array}{llll} & [a]\,[b] & & \\ & [ab] \quad + [c] & & [e] + [d] \\ [ab + c] & & [e + d] & \\ [(ab + c)\,(e + d)] & & & . \end{array}$$

After simplification we obtain the equation

$$x = e + d + \overline{c}(\overline{a} + \overline{b}), \tag{5.1.2}$$

where x can be replaced by the variables d, e, c and a.
Using Table 5.1.2, find equations for subjects d, e, c and a:

$$d = 0 + d + \overline{1}(\overline{0} + \overline{0}) = d\,, \tag{5.1.3}$$

$$e = e + 1 + \overline{1}(\overline{0} + \overline{0}) = 1\,, \tag{5.1.4}$$

$$c = 0 + 0 + \overline{c}(\overline{1} + \overline{1}) = 0\,, \tag{5.1.5}$$

$$a = 0 + 0 + \overline{0}(\overline{a} + \overline{1}) = \overline{a}\,. \tag{5.1.6}$$

It follows from these equations that d is in a state of free choice, e is in the active state, c is in the passive state, and a is in a state of frustration. □

5.2. Second theorem on variety

Formulation: There is a group of subjects with a decomposable graph, such that the group contains a subject capable of being in any one of the four states, depending on the influences exerted by other subjects.
Proof. Let the relation table be as shown in Table 5.2.1, and the combination of influences on subject a by b, c, d, e, f be as shown in Table 5.2.2.

Table 5.2.1
Table of relations

	a	*b*	*c*	*d*	*e*	*f*
a		+	+	+	.	.
b	+		.	.	.	.
c	+	.		+	.	.
d	+	.	+		.	.
e	.	.	.	.		+
f	.	.	.	.	+	

Table 5.2.2
Four sets of influences

	1	2	3	4
b	1	1	1	0
c	0	1	1	1
d	0	0	1	1
e	1	1	0	0
f	0	1	0	0

Using Table 5.2.1, construct the relation graph (Fig. 5.2.1).

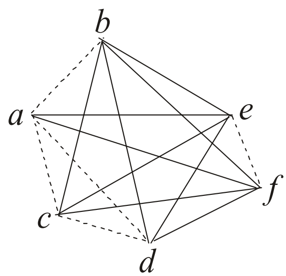

Fig. 5.2.1. Graph of relations

This graph corresponds to the following polynomial:

$$(a + b\ (c + d))\ (e + f) \tag{5.2.1}$$

and to the following diagonal form:

$[c] + [d]$
$[b]\ [c + d]$
$[a] + [b\ (c + d)]$ $[e]+[f]$
$[a + b\ (c + d)]$ $[e+f]$
$[(a + b\ (c + d))\ (e + f)]$ (5.2.2)

We obtain the equation

$$a = (a + b(c + d))(e + f) + \bar{a}\bar{b} . \tag{5.2.3}$$

By substituting the values of variables from the columns of Table 5.2.2 into equation (5.2.3), we obtain:

$$a = a, \quad a = 1, \quad a = 0, \quad a = \bar{a}.$$

These equalities prove the theorem. □