Chapter 6
Extension of the initial model

Within the framework of the initial model, we assume that the relations graph is decomposable and that the set of actions is the same for all subjects. In the extended model these limitations are removed. We generalize the model by assuming that the graph of relations may be different for each subject. Finally, we introduce the concept of being *unaware*. To include this concept in the model, we must suppose that variables may have different values on the first and subsequent tiers of the diagonal form.

6.1. Non-decomposable graph of relations

Since the subjects in a group may have arbitrary relations, the graph for a group containing more than three subjects may be non-decomposable. We suppose that, in this case, the subject excludes other subjects from consideration one by one until the graph of relations becomes decomposable. This moment will necessarily arrive, because for three subjects a graph is always decomposable. We assume that each subject has a strict ordering of the other subjects' importance. First, the subject removes the member of the group that is the least important for him. If after that the graph becomes decomposable, the procedure ends. If the graph is still not decomposable, the least important member out of those who left is removed, and so on until the graph becomes decomposable. Note that different subjects may have different orders of importance.

Consider the graph in Fig.6.1.1. This graph is not decomposable, because its subgraph $<a, b, c, d>$ is $S_{(4)}$. Let the importance of other subjects for subject a decrease in the following order:

$$c, b, d, e, \qquad (6.1.1)$$

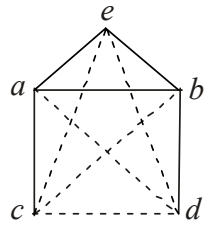

Fig. 6.1.1. Non-decomposable graph

i.e., the most important for *a* is subject *c*, and the least important is subject *e*. The process of removal begins with subject *e*, after which the graph appears as follows (Fig.6.1.2):

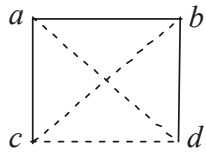

Fig. 6.1.2. Non-decomposable graph

This graph is $S_{(4)}$, so, it is not decomposable. Among the subjects that remain, the least important for *a* is *d*. The graph that appears after *d*'s removal contains three nodes (Fig.6.1.3):

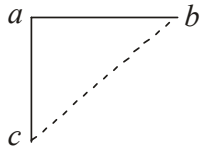

Fig. 6.1.3. Decomposable graph

This graph is decomposable. It corresponds to the polynomial

$$a(b + c).$$

For subject *c*, let the importance of other subjects decrease in this order:

$$e,\ a,\ d,\ b. \qquad (6.1.2)$$

Removal begins with subject b, after which we obtain graph (6.1.4):

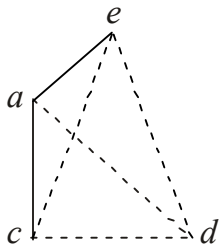

Fig. 6.1.4. Decomposable graph

This graph is decomposable, because there is no $S_{(4)}$ among its subgraphs. It corresponds to the polynomial

$$d + a(e + c).$$

We see that for the list in (6.1.1), subject a needs two removals to transform a non-decomposable graph into a decomposable one. But subject c with list (6.1.2) needs only one removal.

6.2. Individual sets of actions

Every subject may have his own individual set of actions with given realization relations. Consider a group of three subjects: a, b and c.

Subject a. Let this subject be able to perform only one action, α_1. The set of his alternatives is

$$1 = \{\alpha_1\},$$
$$0 = \{\ \}.$$

Subject b. This subject can perform three actions: α_2, α_3, α_4. The set of alternatives is:

$$
\begin{aligned}
1 = &\{\alpha_2,\ \alpha_3,\ \alpha_4\}\\
&\{\alpha_2,\ \alpha_3\}\\
&\{\alpha_3,\ \alpha_4\}\\
&\{\alpha_2,\ \alpha_4\}\\
&\{\alpha_2\}\\
&\{\alpha_3\}\\
&\{\alpha_4\}\\
0 = &\{\ \}
\end{aligned}
$$

Subject c. Let *c* be able to perform two actions, α_5 and α_6. The set of alternatives is

$$
\begin{aligned}
1 = &\{\alpha_5,\ \alpha_6\},\\
&\{\alpha_5\},\\
&\{\alpha_6\},\\
0 = &\{\ \}.
\end{aligned}
$$

This example demonstrates that different subjects may have different sets of alternatives. In modeling choices in such groups, the influences of group members on each subject must relate to an alternative from the particular subject's set of actions.

6.3. Individual graphs of relations

Until this point, we have looked at relation graphs from an external observer's point of view. Let us analyze a more general scheme and assume that the subjects may have different perceptions of the groups content and of relations among its members.

Consider a set of three subjects . Let each of them differently perceive this collection as a group, as in the example in Fig. 6.3.1. Subjects *a* and *c* see all members of the group but have different opinions concerning relations. Subject *b* does not include *a* in the group and perceives his relation with *c* as cooperation, even though *c* believes it is conflict. In this case, the diagonal forms for each subject constructed differently.

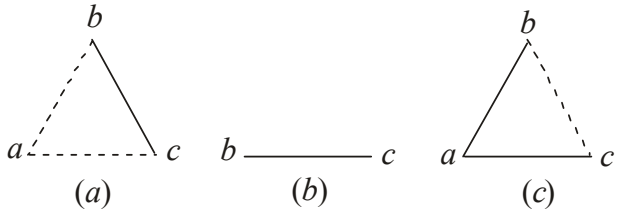

Fig. 6.3.1. Graphs of relations, from the point of view of each subject

6.4. Influences that the subject is aware of and not aware of

Consider a's diagonal form:

$$\begin{matrix} & [a]\ [b] \\ [a\ b] & \end{matrix} \quad . \qquad (6.4.1)$$

The expressions in brackets on the second tier represent two images of the self that the subject has and is aware of. The expression in brackets on the first tier belongs to a domain that the subject is not aware of. The subject becomes aware of the values of variables a and b on the first tier through their appearance on the second tier. Let us take the next step. Suppose that the influences of other subjects on the first and subsequent tiers may differ. So, in the subject's diagonal form, two variables will correspond to each of the other subjects' influences: one is located on the first tier, and the other on all subsequent tiers. But the intention is the same on all tiers. With this extension, the subject corresponds to the following equation:

$$\begin{matrix} & [a]\ [b_2] \\ a = [a\ b_1] & \end{matrix} \quad . \qquad (6.4.2)$$

For example, on the second tier, the one that the subject is aware of, a receives b's direction to choose alternative 1, but on the first tier, the one that the subject is not aware of, a receives

direction to choose 0, thus

$$\begin{matrix} & [a]\ [1] \\ a = & [a\ 0] \end{matrix} \tag{6.4.3}$$

or

$$a = \overline{a} \tag{6.4.4}$$

This means that, as a result of b's influence, subject a will be in a state of frustration.