Chapter 7
Superactivity

The subject will be called *superactive*, if, for all sets of influences from other subjects, the subject chooses alternative 1, i.e., the set containing all actions. A group will be called superactive if every member of the group is superactive.

7.1. Superactive subjects

From the formal point of view, the subject is superactive, if equation

$$a_k = \Phi(a_1, \ldots, a_k, \ldots, a_n) \tag{7.1.1}$$

corresponding to the subject, has only one solution a_k=1 for any set of values of the variables $a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n$. This definition is equivalent to that condition that, for any set of values $a_1, \ldots, a_k, \ldots,$ a_n, the following identity holds:

$$\Phi(a_1, \ldots, a_k, \ldots, a_n) \equiv 1 \ . \tag{7.1.2}$$

Let us prove this statement. It is clear that (7.1.1) follows from (7.1.2). We will show now that (7.1.2) follows from (7.1.1).

Function $\Phi(a_1, \ldots, a_k, \ldots, a_n)$ can be written as

$$\Phi = A\,(a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n)\,a_k + B(a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n)\,\overline{a}_k \ ,$$

that is,

$$a_k = Aa_k + B\overline{a}_k \ . \tag{7.1.3}$$

Since equation (7.1.1) has only one solution a_k=1,

$$A = 1, B = 1 \tag{7.1.4}$$

for any values of variables a_1, ..., a_{k-1}, a_{k+1}, ..., a_n.
Thus, function Φ can be represented as

$$\Phi = a_k + \bar{a}_k \equiv 1,$$

where a_k takes up any value. It follows from this that (7.1.2) is true.

7.2. Superactive groups

Consider the relations graphs in Fig. 7.2.1:

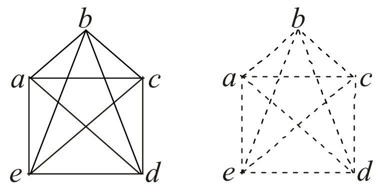

Fig. 7.2.1. Relation graphs of homogeneous groups

The left-hand graph depicts a group in which every two subjects are in a relation of cooperation, and the right-hand graph depicts a group in which every two subjects are in conflict. We will call such groups *homogeneous*. Graphs of homogeneous groups are decomposable; they correspond to polynomials of the type

$$a_1\ a_1\ \ldots\ a_n$$

and

$$a_1 + a_1 + \ldots + a_n,$$

Their two-tier diagonal forms are

$$\begin{matrix} & [a_1]\ [a_2]\ \ldots\ [a_n] \\ \Phi = & [a_1\ a_2 \ldots\ a_n] \end{matrix} \qquad (7.2.1)$$

and

$$[a_1] + [a_2] + \ldots + [a_n]$$

$$\Phi = [a_1 + a_2 + \ldots + a_n] \qquad (7.2.2)$$

or

$$a_1 a_2 \ldots a_n + \overline{a_1 a_2 \ldots a_n} \equiv 1\,, \qquad (7.2.3)$$

and

$$a_1 + a_2 + \ldots + a_n + \overline{a_1 + a_2 + \ldots + a_n} \equiv 1\,, \qquad (7.2.4)$$

respectively. Therefore, every subject of a homogeneous group is superactive, always chooses alternative 1 and cannot choose any other alternative.

The concept of superactivity allows us to understand the behavior of a *crowd*, considered as a homogeneous group, if we suppose that relations between its member are the same. A crowd's behavior is uncontrollable from within, that is, no subject inside the crowd can influence the behavior of surrounding subjects. This characteristic is described by (7.2.3) and (7.2.4). No subject's choice depends on any other subjects' influences; every subject always chooses alternative 1.

There are other groups, aside from homogeneous groups, that can also be superactive. Consider the graph in Fig. 7.2.2:

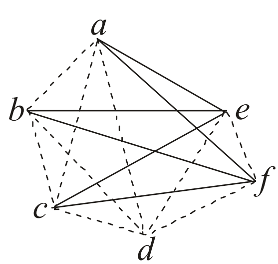

Fig. 7.2.2. Relation graph

This graph is decomposable. It corresponds to the polynomial

$$d + (a + b + c)\,(e + f) \tag{7.2.5}$$

and the diagonal form

$$\begin{array}{l} \qquad\qquad\qquad\qquad [a] + [b] + [c] \qquad\qquad [e] + [f] \\ \qquad\qquad\qquad [a + b + c] \qquad\qquad\quad [e + f] \\ \qquad [d] + [(a + b + c)\,(e + f)] \\ [d + (a + b + c)\,(e + f)] \end{array} \tag{7.2.6}$$

which can be represented as

$$d + (a + b + c)(e + f) + \overline{d + (a + b + c)(e + f)} \equiv 1 \ .$$

Thus, the group is superactive, although it is not homogeneous.

If the graph of relations is unchanged for long time, then, independently of the subjects' changing influences upon one another, superactivity is a state of long duration.

7.3. Theorem on the impossibility of superpassivity

Let us pose a question: Is there a subject who chooses 0 under all sets of influences from other subjects? Such a subject could be called *superpassive*. For such a subject to exist, an equation of the type (7.1.1) must have the solution $a_k = 0$ for any set of values of the variables $a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n$.

Theorem on the impossibility of superpassivity. For any $n \geq 2$ there is at least one set of values of the variables $a_1, \ldots, a_{k-1}, a_{k+1}, \ldots, a_n$, for which an equation of the type (7.1.1) has the solution $a_k = 1$.

Proof. Function Φ corresponding to any non-elementary diagonal form can be represented as

$$\Phi = P(a_1, \ldots, a_k, \ldots, a_n) + \overline{W}(a_1, \ldots, a_k, \ldots, a_n) \ . \tag{7.3.1}$$

Function P is a polynomial with operations $\cdot$ and $+$, but without the unary operation of negation. If every variable in (7.3.1) is equal to 1, then P =1, from which

$$\Phi(1, \ldots, a_k=1, \ldots, 1) = 1,$$

i.e., an equation of the type (7.1.1) has solution a_k=1 at least for one set of the variables' values.

Therefore, no superpassive subjects can exist. □