Appendix

Problems and Exercises

Introduction

The brief descriptions of subjects' behavior are given below. In each case determine whether the subjects' actions satisfy the anti-selfishness principle.

1. In 356 b. a. Herostratus sets fire to the temple of Artemis in his quest for fame.

2. A rich man leaves his fortune to the church in hope of getting to heaven.

3. Children ask their father to come home early from work. The father comes late because he stopped at a bar.

4. The people of a kingdom ask the king to declare amnesty, but the king remains unbending in spite of the risk of uprising.

5. Danko, a legendary hero, tears out his heart to light the way through forest for his people.

6. People ask for bread and circuses from the ruler, but he gives them only circuses, because he too loves them much.

7. In spite of the spectators' protests, the emperor pardons a defeated gladiator because he does not want to kill people.

8. The next time he pardons a defeated gladiator, the emperor tells the spectators that anyone who desires death for another will go to hell. The silence shows that the people understand.

Chapter 1

I. Draw Venn diagrams for the following sets, assuming that they have non-empty intersections.

1. $A + BC$ 2. $(A + B)C$ 3. ABC 4. $A + B + C$

5. $\overline{A + B}$ 6. $\overline{AB}$ 7. $A + \overline{B}$ 8. $A\overline{B}$

9. $\overline{A} + \overline{B}$ 10. $\overline{\overline{A} + \overline{B}}$ 11. $\overline{\overline{A}\,\overline{B}}$ 12. $\overline{\overline{A} + B}$

II. Analyze each of the following equations:
(1) determine whether it can be solved
(2) if it is, indicate an interval

13. $x = (a + b)x + b\overline{x}$

14. $x = bx + (a + b)\overline{x}$, where $(a + b) \supset b$

15. $x = x + a\overline{x} + b$ 16. $x = x + b$ 17. $x = \overline{x} + c$

III. Analyze the equations defined on the set of all subsets of the universal set $\{\alpha, \beta, \gamma\}$:

18. $x = \{\alpha, \beta\}\, x + \{\alpha\}$ 19. $x = \{\beta\}\, x + \{\alpha, \beta\}\, \overline{x}$

20. $x = \{\alpha\}\, x + \{\alpha\}\, \overline{x}$ 21. $x = \overline{\{\alpha\}}\, x + \{\alpha, \beta\}\, \overline{x}$

22. $x = \overline{\{\alpha\}}\, x + \{\beta, \gamma\}$ 23. $x = \overline{\{\alpha, \beta\}}\, x + \{\alpha, \beta\}$

24. $x = \{\alpha\}\, \overline{x}$

IV. Write the following exponential formulas in the linear form:

25. a^{b^c} 26. $a + b^{c^d}$ 27. $a^{b+c} + d^e$ 28. $a^{b^{c+d}}$

29. a^{b+a^c} 30. $a^{b^{a+b^{\overline{a}}}}$ 31. $a^{ab^{\overline{c}}}$ 32. $a^{\overline{b}a^b}$

33. $a^{\overline{b}^{\overline{c}}+c^{\overline{b}}}$ 34. $a^{0^{\overline{b}+0^{\overline{a}}}}$

V. Analyze the equations given in exponential form:

35. $x = a^x$ 36. $x = x^x$ 37. $x = x^{x^x}$

38. $x = a^x + b^x$ 39. $x = a^x + b$ 40. $x = a^x + x$

41. $x = a^{b^x}$ 42. $x = (a + b)^{x+c}$ 43. $x = (ax)^{bx}$

44. $x = (a + x)^{b+x}$ 45. $x = x^{a^x}$ 46. $x = x^{a+x^b}$

VI. Analyze the equations defined on set M of all subsets of the universal set $\{\alpha, \beta, \gamma\}$:

47. $x = 0^{\{\alpha,\beta\}\overline{x}}$ 48. $x = \{\alpha,\beta\}^{\overline{\{\gamma\}}+\overline{x}}$

49. $x = \{\alpha\}^{\{\alpha,\gamma\}+\overline{\{\beta\}}x}$ 50. $x = \{\alpha,\gamma\}^{\{\beta\}+\{\alpha\}x}$

Chapter 2

I. We will designate the solid sides by R, and the broken ones by $\overline{R}$. Consider graph

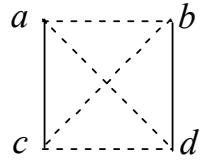

Fig. A1-1. Stratifiable graph

This graph can be stratified. It can be divided into two minimal strata on $\overline{R}$:

$$<a, c> \overline{R} <b, d>$$

In a similar way, divide the following graphs into minimal strata:

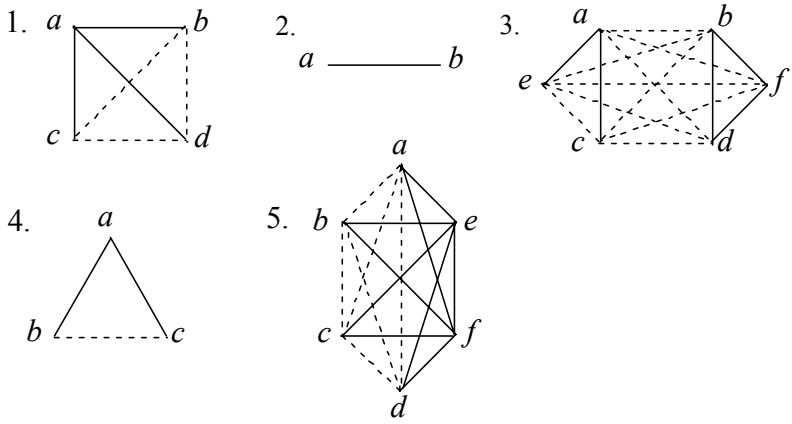

Fig. A1-2. Problems 1-5

II. For each graph, determine whether it is $S_{(4)}$:

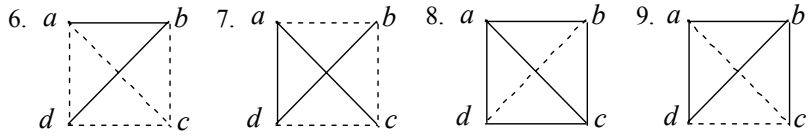

Fig. A1-3. Problems 6-9

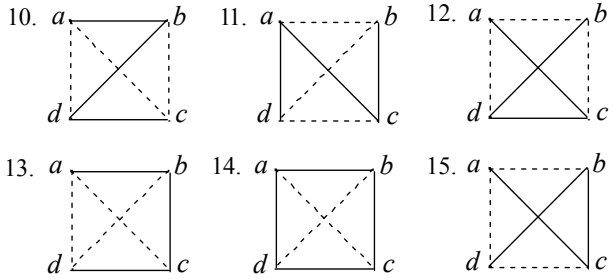

Рис. A1-4. Problems 10-15

III. Determine whether each graph contains subgraphs $S_{(4)}$, and if so, how many:

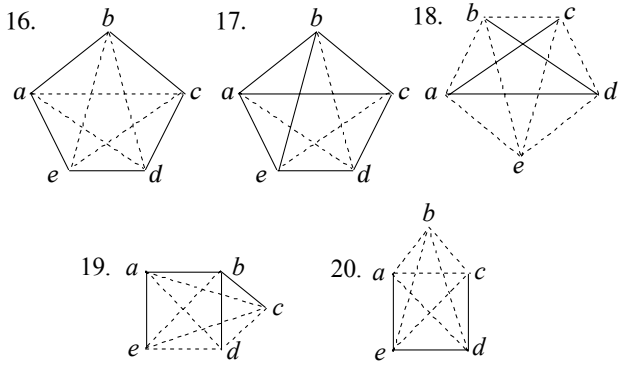

Fig. A1-5. Problems 16-20

Chapter 3

I. Determine whether each graph can be decomposed:

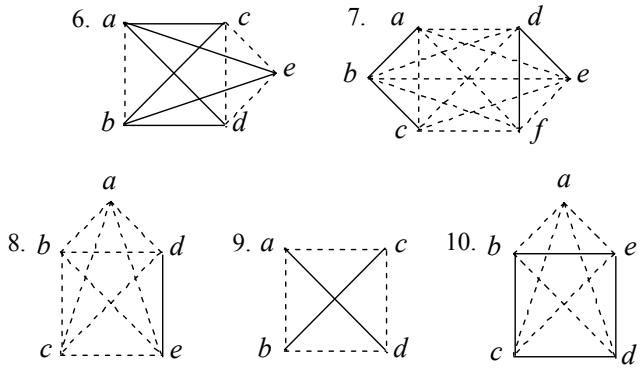

Fig. A1-6. Problems 1-5

II. Write polynomials for the following decomposable graphs:

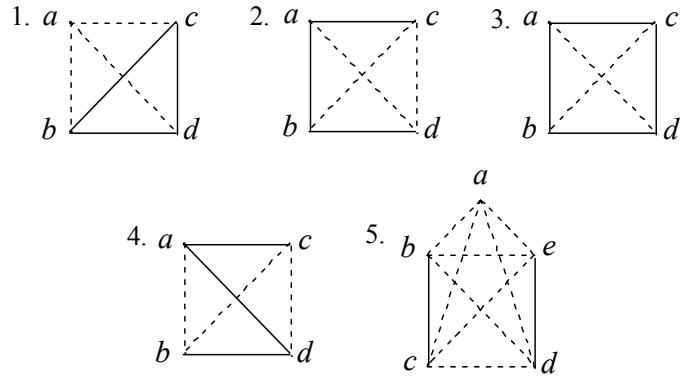

Fig. A1-7. Problems 6-10

III. Draw graphs corresponding to the following polynomials:

11. $a(b + c)$ 12. $a + bc$ 13. $a + b + c$

14. abc 15. $ab + cd$ 16. $ab + c\,(d + e)$

17. $(a + bc)\,d$ 18. $a + b + cd$ 19. $(a + b)(c + d)$

20. $(a + b)(c + de)$ 21. $(ab + c)de$

IV. Write diagonal forms for the following polynomials:

22. $a + b + c$ 23. $a + bc$ 24. $(a + b)(c + d)$

25. $a(b + c)$ 26. $d + a(b + c)$ 27. $abc\,(d + e)$

28. $a\,(b + c) + de$ 29. $(ab + cd)\,e$ 30. $ab\,(c + d)$

31. $a + bc + def$ 32. $a + b(c + d\,(e + f))$

33. $a + bc\,(d + ef)$ 34. $(a + b + c)\,(d + ef)\,g$

35. $(ab + cd)\,(eh + fg)$ 36. $abc + d\,(g + ef)$

Chapter 4

I. Taking Fig. 4.2.3 as an example, draw diagrams of partial order for the following diagonal forms

1. $[a + b]$

$[a] + [b]$

2. $[ab\ (c + d)]$

$[a]\ [b]\ [c + d]$

$[c] + [d]$

3. $[(a + bc)\ d]$

$[b]\ [c]$

$[a] + [bc]$

$[a + bc]$

$[d]$

II. Transform each of the following diagonal forms into linear notation (remember that diagonal forms are exponential expressions in which parentheses and brackets are equivalent):

4. $[a + bc]$

$[b]\ [c]$

$[a] + [bc]$

5. $[a\ (b + c)]$

$[b] + [c]$

$[a]\ [b + c]$

6. $[a\ (bc + d)]$

$[b]\ [c]$

$[bc] + [d]$

$[a]\ [bc + d]$

7. $[a + b\ (c + d)]$

$[c] + [d]$

$[b]\ [c + d]$

$[a] + [b\ (c + d)]$

III. Analyze equations of the type $a = \Phi(a)$, where $\Phi(a)$ is as follows:

$$[a]\,[b] \qquad [c]\,[d]$$
$$[ab] \qquad + [cd]$$
8. $[ab + cd]$

$$[b] + [c] + [d] + [e]$$
$$[a]\,[b + c + d + e]$$
9. $[a\,(b + c + d + e)]$

$$[b]\,[c] \qquad\qquad [e]\,[f\,]$$
$$[a] + [bc] \qquad\qquad [d] + [ef\,]$$
$$[a + bc] \qquad\qquad [d + ef\,)]$$
10. $[(a + bc)\,(d + ef\,)]$

IV. The graph of relations between subjects a, b, c and d is given in Fig. A1-8:

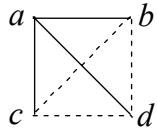

Fig. A1-8. Relations graph for problem 11

The universal set of actions is $1 = \{\alpha, \beta, \gamma, \delta\}$, and the influence matrix is as follows:

Table A1-1. Matrix of influences for problem 11

	a	b	c	d
a	a	$\{\beta\}$	$\{\alpha,\beta\}$	1
b	$\{\alpha\}$	b	$\{\alpha\}$	$\{\beta\}$
c	$\{\alpha\}$	$\{\alpha\}$	c	$\{\beta\}$
d	$\{\alpha,\beta\}$	$\{\alpha\}$	$\{\beta\}$	d

11. Find the choice of each subject

V. Consider the situation described in section 4.5, assuming that subject a_1 chooses alternative 1. The universal set is $\{\alpha, \beta, \gamma, \delta\}$.

12. Find the set of actions attractive for the group (P).

13. Find the set of actions attractive for subject a_1 (W).

14. Find the set of actions prohibited for subject a_1 $(\overline{P}W)$.

VI. For a subject with universal set $1 = \{\alpha, \beta, \gamma\}$ find the set of actions that are always chosen (R) and the set of actions that are never chosen (S), if

15. $A = \{\alpha, \beta\}$, $B = 0$; 16. $A = 1$, $B = \{\alpha, \beta\}$;

17. $A = 1$, $B = 0$; 18. $A = 0$, $B = 0$

Chapter 5

I. Using the graph of relations in Fig. 5.1.1, find:

1. The state of subject *b* with the matrix of influences in Table 5.1.2

2. The states of subjects *a*, *b*, *c*, *d*, *e* with the following matrix of influences:

Table A1-2. Matrix of influences for problem 2

	a	*b*	*c*	*d*	*e*
a	*a*	1	0	0	0
b	0	*b*	0	0	1
c	0	1	*c*	1	1
d	1	1	0	*d*	0
e	0	0	0	0	*e*

II. Determine whether subjects *a*, *b*, *c*, *d*, *e*, corresponding to equations (5.1.2), can be all at the same time

3. in the active state

4. in the passive state

5. in a state of frustration

6. in a state of free choice

III. The graph of relations is as follows:

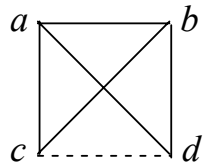

Fig. A1-9. Graph of relations for problems 7 and 8

7. Construct the matrix of influences for which subjects *a* and *b* would be in a state of frustration, and subjects *c* and *d* in a state of free choice.

8. Is it possible to change only one relation in such a way that all group members would be in the active state, independently of the matrix of influences?

Chapter 6

I. A graph of relations is as follows:

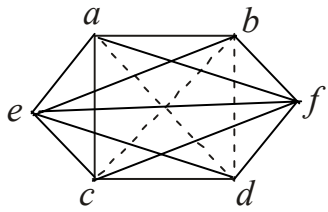

Fig. A1-10. Graph of relations for problems 1-4

Find the possible choices for subject *a* if the order of other subjects' significance is:

1. *b*, *c*, *d*, *e*, *f* 2. *b*, *f*, *e*, *d*, *c*

3. *d*, *b*, *c*, *f*, *e* 4. *f*, *d*, *e*, *c*, *b*

II. Consider a group with the graph of relations in Fig. A1-11:

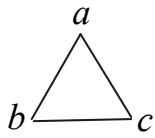

Fig. A1-11. Graph of relations for problems 5 and 6

The universal set for *a* is $\{\alpha\}$, for *b* $\{\beta, \gamma\}$, for *c* $\{\delta, \eta, \theta\}$.

5. What is each subject's choice?

6. What alternatives can they realize, if action β is incompatible with γ, and action δ is incompatible with η?

Chapter 7

I. The following figures show graphs or relations between subjects, but the relation between *a* and *b* is omitted. What should this relation be (cooperation or conflict) to make the group superactive?

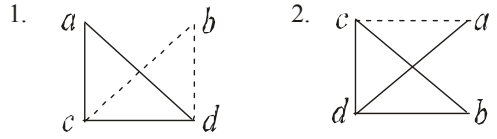

Fig. A1-12. Problems 1 and 2

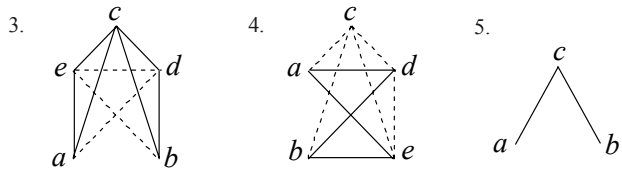

Fig. A1-13. Problems 3-5

II. What should be the relation of *a* and *b* so that the group does not become superactive?

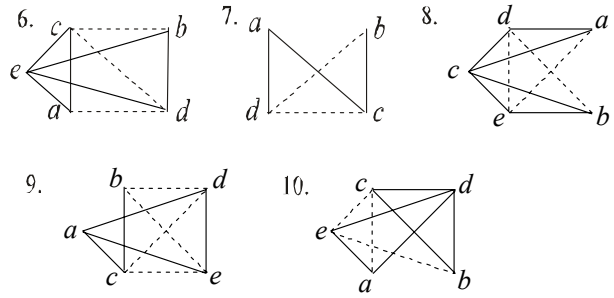

Fig. A1-14. Problems 6-10

Chapter 8

I. Consider a group with the relations given in Fig. A1-15:

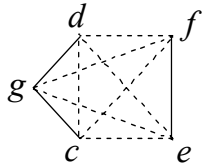

Fig. A1-15. Relations graph for problems 1-4

1. Is this group superactive?

2. Will this group be superactive if a peacemaker *a* joins it?

3. Will this group be superactive if two peacemakers *a* and *b*, who are in union, join the group?

4. Will this group be superactive if two peacemakers *a* and *b*, who are in conflict, join the group?

II. Consider the graph of relations

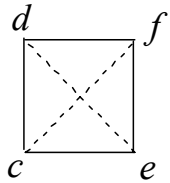

Fig. A1-16. Relation graph for problems 5-7

5. Is this group superactive?

6. Will this group be superactive if subject *a*, who is friendly with everyone else, joins the group?

7. Will this group be superactive if subject *a*, who is in conflict with everyone else, joins the group?

Chapter 9

I. Manipulation through influence

Scheme:

"a wants b to choose x; to achieve this a exerts influence x"

Consider the graph

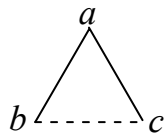

Fig. A1-17. Relations graph for problems 1-8

Can a apply the given scheme to incline b to choose 1, if

1. $c = 1$ 2. $c = 0$ 3. $d = 1$ 4. $d = 0$?

Can a apply the given scheme to incline b to choose 0, if

5. $d = 0$ 6. $c = 1$ 7. $d = 0, c = 0$ 8. $d = 0, c = 1$

Scheme (inverse control):

"a wants b to choose x, but to achieve this a has to exert influence $\bar{x}$ "

Consider the relations

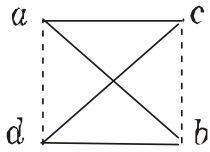

Fig. A1-18. Relations graph for problem 9

9. Let a want b to choose 1. Can he achieve this by using inverse control?

Scheme:

"a wants b to have freedom of choice and exerts influence x to achieve this"

Consider the graph of relations

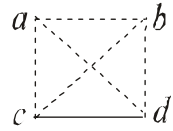

Fig. A1-19. Relations graph for problems 10-13

Can a move b into a state of free choice if

10. $c = 1$, $d = 1$; 11. $c = 0$, $d = 1$;

12. $c = 1$, $d = 0$; 13. $c = 0$, $d = 0$.

Scheme:

"a wants b to become incapable of making a choice and exerts influence x"

Consider the graph

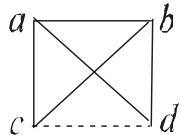

Fig. A1-20. Graph of relations for problems 14-17

Can a fulfill his task if

14. $c = 0$, $d = 0$; 15. $c = 1$, $d = 1$;

16. $c = 0$, $d = 1$; 17. $c = 1$, $d = 0$?

II. Manipulations with relations

Scheme:

"*a* wants *b* to have a chance to obtain the freedom of choice and changes relation (*a*, *b*)"

Let the graph of relations be as follows:

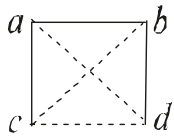

Fig. A1-21. Relations graph for problems 18 and 19

18. Will *a* achieve the goal by changing the relation (*a*, *b*)?

Scheme:

"*a* wants *b* to have a chance of obtaining freedom of choice and leaves the group"

19. Will *a* reach his goal if he leaves the group whose relations are given in Fig. A1-21?

III. Manipulation through the order of subjects' significance

Scheme:

"*a* does not want to be removed from *b*'s graph of relations and changes the order of subjects' significance for *b*"

Consider non-decomposable graph of relations:

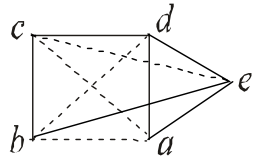

Fig. A1-22. Graph of relations for problems 20-22

Will *a* remain in the graph for the following orders of subjects' significance:

20. *c*, *d*, *a*, *e* ; 21. *e*, *c*, *a*, *d* ; 22. *d*, *e*, *a*, *c*

Chapter 10

I. The son corresponds to the following graph of relations:

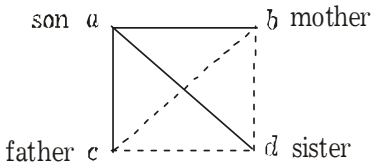

Fig. A1-23. Relations graph for problems 1-3

The son's universal set is $\{\alpha, \beta, \gamma\}$.

1. What will be the son's choice if $b = \{\alpha\}$, $c = \{\alpha\}$, $d = \{\alpha\}$?

2. What influence must the mother exert to make the son marry?

3. In what state will the son be, if mother, father, and a sister incline him not to marry?

II. Consider the graph in Fig.10.2.1. Let John and Tom be moved to another cell. Everyone else insists on his own plan of escape, and John's and Tom's plans are not considered.

4. What are choices made by Bob, Peter, and Larry?

III. Consider the scenario with the lost cigarette (Fig.10.3.1). Tom is moved to the prison hospital, but John continues to suspect him. Relations between John, Bob, and Peter remain the same. All three of them sympathize with Tom and feel solidarity with him.

5. What is John's choice?

IV. Consider the graph in Fig.10.4.1.

6. What is Edward's choice if the significance order of his group members is as follows: Alex (a), Bart (b), David (d), Gregory (g), and these influences do not change?

Chapter 11

I. Consider the scenario corresponding to Fig. 11.1.1.

1. The conflict between the political elite and the military has changed to cooperative relations and no further changes occurred. What will be the choice of the political elite?

2. The secret police are disbanded, but no other changes take place. What will be the choice of the political elite?

3. The military obtain power. The population is passive ($d = 0$). The political elite exists as an active force. What decision will be made by the military?

II. Consider the graph in Fig. 11.2.1.

4. Parties f and g are disbanded, in addition; also, $d = 0$ and $e = 0$. What choice will the President make?

5. A conflict takes place between the President and party c (Fig.11.2.1). Also, $b = \{\alpha\}$, $c = \{\alpha\}$, $g = 0$. What choice will be made by the President?

6. The President is overthrown. Party g takes power. There is discussion of war with a neighboring country. The set of alternatives is $\{1, 0\}$, where 1 means war and 0 means peace, $c = 0$, $b = 0$, $d = 0$, $e = 1$, $f = 1$. In what state is party g?

III. Consider the scenario corresponding to Fig. 11.3.1.

7. The undercover police agents succeed in making gang a break with gangs b and e. Will the group cease to be superactive?

Chapter 12

I. Consider the scenario for Fig. 12.1.1. Let the matrix be as follows:

Table A1-3. Matrix of influences for problems 1 and 2

		a	*b*	*c*	*d*
USA	*a*	*a*	1	1	0
Soviet Union	*b*	1	*b*	0	1
England	*c*	0	0	*c*	1
Germany	*d*	0	1	0	*d*

1. Suppose that the conflict between Germany and England has ended. In what state will each of the four countries be?

2. Suppose that the conflict between USA and Germany has also ended. In what state will each country be?

II. Consider the graph of relations in Fig.12.2.1.

3. I what state would each country be, if the Soviet Union and the USA were in cooperation at that time?

III. Consider the relations graph in Fig. 12.3.1.

4. Let Russia be in cooperation with Israel and the USA. In what state would each subject be?

IV. The conflict between China and Taiwan has persisted for several decades. During this time, the USA and Russia have had the greatest influence on the situation. We will consider that China and Russia are in cooperation, as are Taiwan and the USA; a conflict exists between these two pairs. Let the matrix of influences be as follows:

Table A1-4. Matrix of influences for problem 5

		a	*b*	*c*	*d*
China	*a*	*a*	0	0	0
Russia	*b*	0	*b*	0	0
Taiwan	*c*	1	0	*c*	1
USA	*d*	1	0	0	*d*

5. In what state is each subject?

Chapter 13

I. Consider the scenario corresponding to Fig. 13.1.1. Suppose platoon a goes over to the side of the battalion (d). The relation between a and d changes from conflict to cooperation, and the relation between a and b and c changes to conflict.

1. What will the battalion commander choose?

II. In some country there is a civil war in progress. Tribe b intends to destroy tribe d. A third tribe, c, is cooperative with b and in conflict with d. There is a peacemaker a in the country. He persuaded the leaders of the tribes c and d try to persuade tribe b to cease hostilities, by allowing b to fish in their lakes. The set of alternative for b is $\{1, 0\}$, where 1 is fighting and 0 is to stop fighting. Permission to fish in the lakes belonging to tribes c and d is their influence on tribe b, inclining b to choose stop fighting: $c = 0$, $d = 0$.

2. What else must the peacemaker do to stop the bloodshed?

III. Three cooperative countries, a, b and c have a potential enemy, country d. Countries a, b and c plan to start a common project: construction of a new military aircraft. There are two types to choose from: α and β. Plane α has better battle qualities, but is more expensive than β. Country a prefers α, and country b prefers β. Each of them tries to persuade country c to support its project. In addition, it is known that the potential enemy, country d, has begun developing project α. For country c, this is an additional argument in favor of α.

3. What plane will c support?

Chapter 14

I. Consider the trial relations represented by the graph in Fig. 14.1.2. A public prosecutor is included in the trial and is in conflict with all participants of the trial.

1. Will the trial remain perfect?

II. A public defender is included in the trial. He is conflict only with the prosecutor.

2. Will the trial remain perfect?

III. Let a conflict appear between the judge and the defendant during the trial, and let the defendant be removed from the hall.

3. Will the trial remain perfect?

Conclusion

1. Analyze the situation with two friends described in the Conclusion.